Technische Universität München (I7) **Winter Semester 2021-22**
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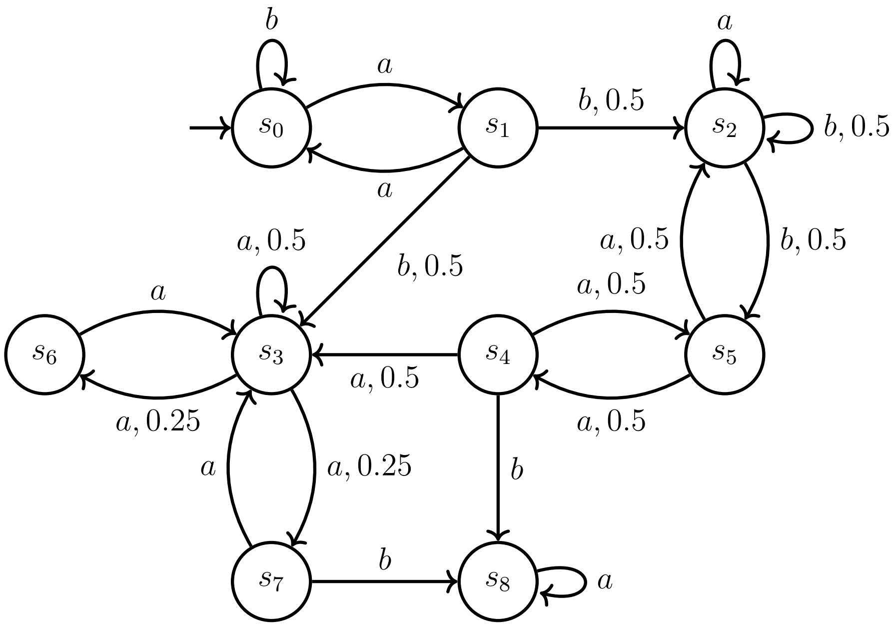
Quantitative Verification – Exercise sheet 11

Exercise 11.1
Compute the MECs (both states and actions) of the following MDP.



Exercise 11.2
On Figure 1, compute $\mathtt{R}_{=?}[\mathtt{C} \leq 4]$ and check whether $\mathtt{R}_{\geq 1}[\mathtt{C} \leq 2]$ holds. Also, compute $\mathtt{R}_{=?}[\mathtt{I} = 4]$.

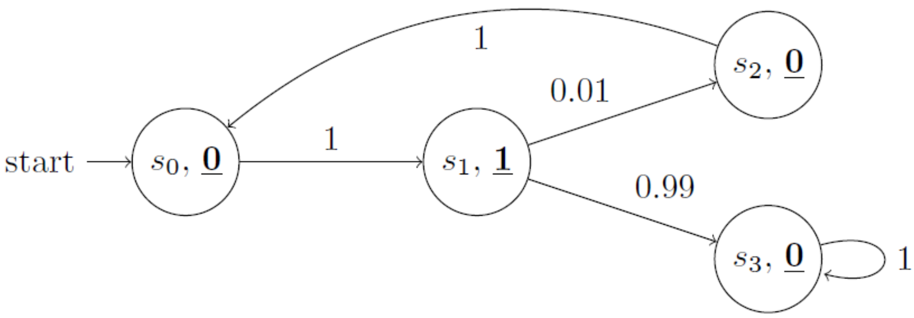


Figure 1: State rewards are bold-faced and underlined.

Exercise 11.3

The *instantaneous reward* of a path at time t associates with a path, the reward in the state of that path when exactly t time units have elapsed. In general, *instantaneous reward* refers to the expected reward of a model at a particular instant in time. See lecture slides for more details. Are memoryless schedulers sufficient to obtain optimal instantaneous rewards? If yes, give a proof sketch. If no, give a counterexample.

Exercise 11.4

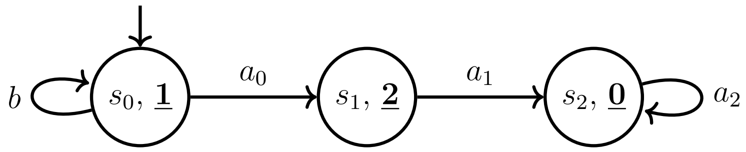
We have seen expected step-bounded reward and expected long-run average reward. How can you rephrase (bounded) reachability as an instance of these problems?

Solution 11.1
MECs:

- $(\{s_0, s_1\}, \{(s_0, a), (s_0, b), (s_1, a)\})$
- $(\{s_2\}, \{(s_2, a)\})$
- $(\{s_3, s_6, s_7\}, \{(s_3, a), (s_6, a), (s_7, a)\})$
- $(\{s_8\}, \{(s_8, a)\})$

Solution 11.2
R=1 for $\mathtt{C} \leq 4$, and $\mathtt{R}_{\geq 1}[\mathtt{C} \leq 2]$ holds because R=1 for $\mathtt{C} \leq 2$. R=0.01 for $\mathtt{I} = 4$.

Solution 11.3



MDP which shows that memoryless schedulers do not exist for instantaneous rewards. (e.g. take b twice and a_0 to maximize instantaneous reward in 3 steps)

Solution 11.4

- Bounded reachability: Make target states absorbing, i.e. remove all outgoing transitions and add a self-loop, set their reward to 1 and all other rewards to 0.
- Unbounded reachability: Cannot be phrased easily as bounded reward problem. For unbounded, apply the above idea.