Technische Universität München (I7) **Winter Semester 2021-22**
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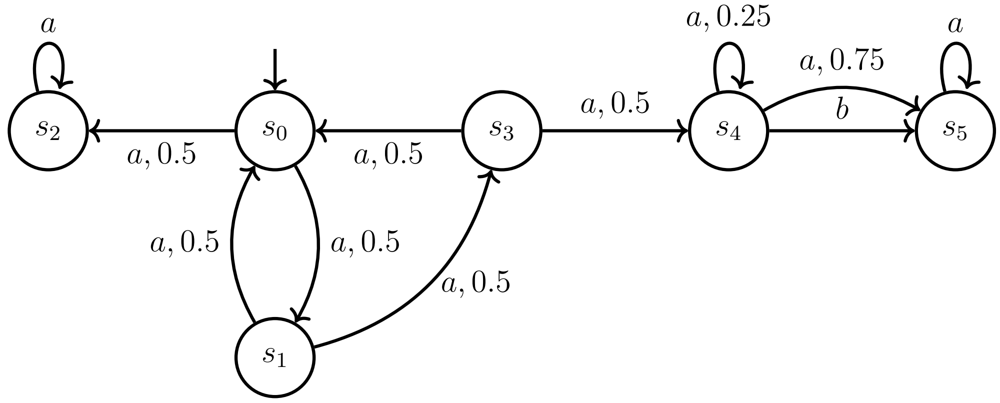
Quantitative Verification – Exercise sheet 10

Exercise 10.1
Consider the following MDP.



Write down the reachability LP for $B = \{s_5\}$.

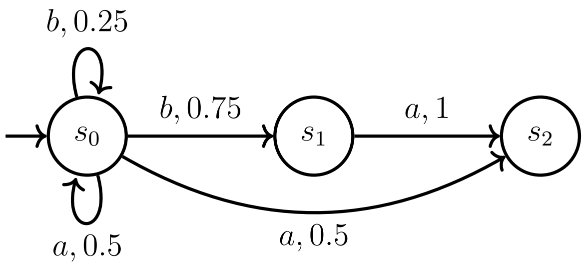
Exercise 10.2
From the slides, we know that memoryless schedulers are sufficient for (unbounded) reachability, but not for its bounded counterpart. Try to come up with an MDP where some bounded reachability query can only be maximized by a scheduler with memory. Argue why finite memory is sufficient. How would you solve a bounded reachability query in general?

Exercise 10.3
We defined the "stopping criterion" of value iteration as $\max_s |x_{n+1}(s) - x_n(s)| < \epsilon$ for some small ϵ. This tends to work in practice, but is not sound in general, i.e. there are some MDPs where this property is fulfilled, but the resulting strategy is not optimal. Can you think of such an MDP?

Solution 10.1
$S^{max}_{>0} = \{s_5, s_4, s_3, s_1, s_0\}$, hence $S^{max}_0 = \{s_2\}$. We have that $S\backslash(B \cup S^{max}_0) = \{s_0, s_1, s_3, s_4\}$. Then, we can derive the LP as follows:

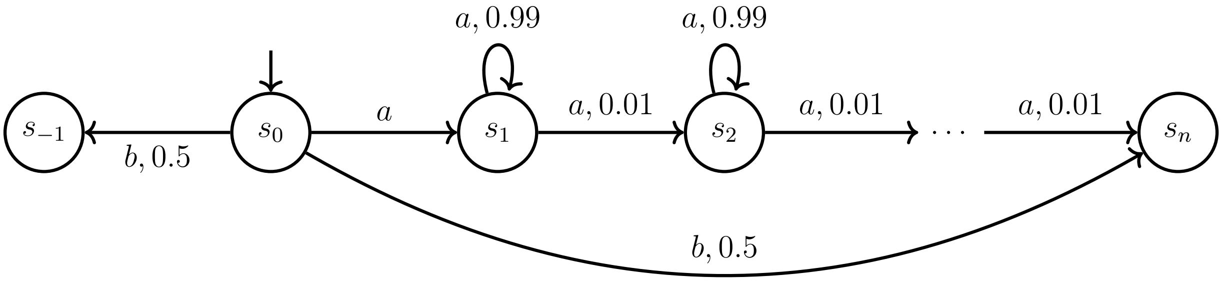
$$\begin{aligned} \text{minimize } & \sum_{s \in S} x(s) \\ \text{where, } & x(s_5) = 1 \\ & x(s_2) = 0 \\ & x(s_0) \geq 0.5 \cdot x(s1) + 0.5 \cdot x(s_2) \\ & x(s_1) \geq 0.5 \cdot x(s0) + 0.5 \cdot x(s_3) \\ & x(s_3) \geq 0.5 \cdot x(s0) + 0.5 \cdot x(s_4) \\ & x(s_4) \geq 0.25 \cdot x(s4) + 0.75 \cdot x(s_5) \\ & x(s_4) \geq x(s_5) \end{aligned}$$

Solution 10.2



Target: $B = \{s_2\}$, Bound: $n = 2$.
The optimal reachability probability of can only be achieved by playing b and then a if remaining in s_0. This yields a total probability of $0.5 + 0.5 \cdot 0.75 = 0.875$, compared to the 0.75 achieved when playing only either a or b. Solving bounded reachability: Value Iteration ($x_n(s) = \sup_\Theta \mathcal{P}^\Theta_s[\mathbf{F}^{\leq 10} B]$) or encoding the step bound into an MDP and solving the unbounded reachability query.

Solution 10.3



To reach $\{s_n\}$, the optimal strategy is to play a everywhere, guaranteeing a probability of 1. Yet, value iteration will eventually stop with a value of 0.5 by playing b in s_0.