

## Quantitative Verification – Exercise sheet 9

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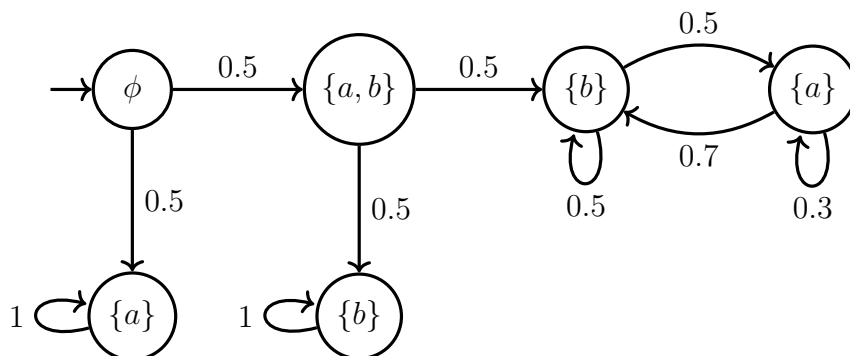
### Exercise 9.1

Draw a Rabin automaton for the following formulae:

- $\mathbf{G} \neg a \wedge \mathbf{G} \mathbf{F} b$
- $\mathbf{G} (a \Rightarrow \mathbf{F} b)$

### Exercise 9.2

Consider the following Markov Chain.



Check whether  $\mathcal{P}[\mathbf{F} \mathbf{G} a \vee \mathbf{F} \mathbf{G} \neg a] = 1$  holds on the given Markov Chain, using the methods from the lecture.

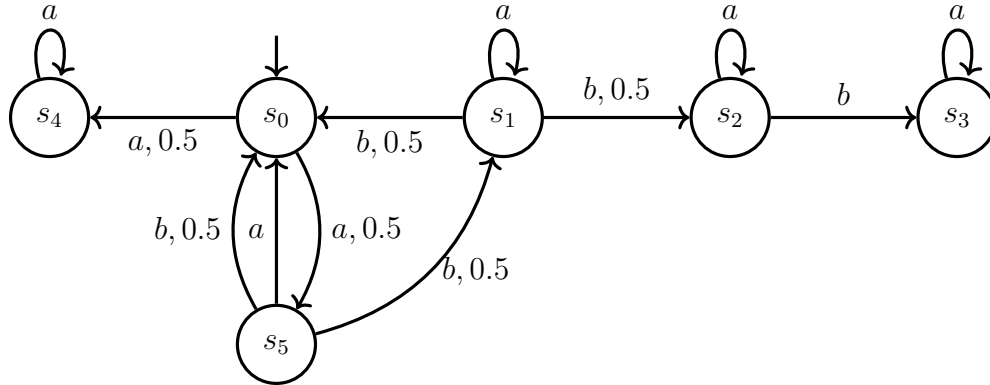
### Exercise 9.3

Model the following situation as an MDP:

*A robot is placed in a  $3m \times 2m$  arena at the south-west corner  $(0, 0)$ . Its goal is to reach a repair station at  $(3, 1)$ , but the arena is not without its challenges. At  $(2, 0)$  and  $(3, 2)$  are two rotor blades spinning at 20000 RPM which is certain death for the robot. A previous close encounter with another such hazard has left the robot with certain eccentricities. Firstly, it cannot move south or west. Secondly, if it tries to move north, it succeeds only 90% of the time. Finally, 10% of attempts to move east is accompanied with a major southwards deviation, which makes the robot end up 1m south of its intended target.*

**Exercise 9.4**

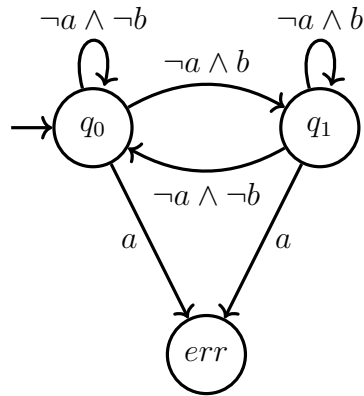
Consider the following MDP.



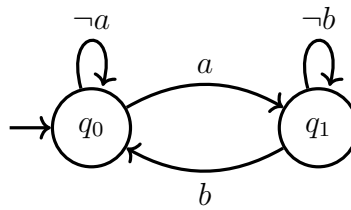
What is the maximal probability of reaching  $s_4$ ? What is the optimal strategy for maximizing the probability of reaching  $s_3$ ? How many optimal strategies are there?

**Solution 9.1**

(a) Rabin automaton for  $\mathbf{G} \neg a \wedge \mathbf{G} \mathbf{F} b$ :

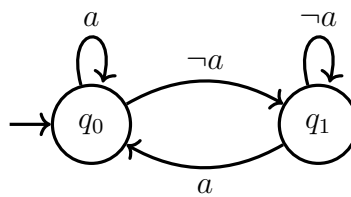


(b) Rabin automaton for  $\mathbf{G} (a \Rightarrow \mathbf{F} b)$ :



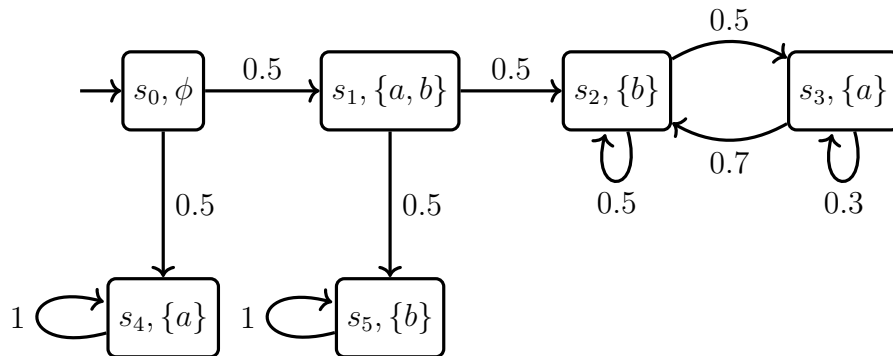
**Solution 9.2**

First, we derive the Rabin Automaton for the given LTL formula:

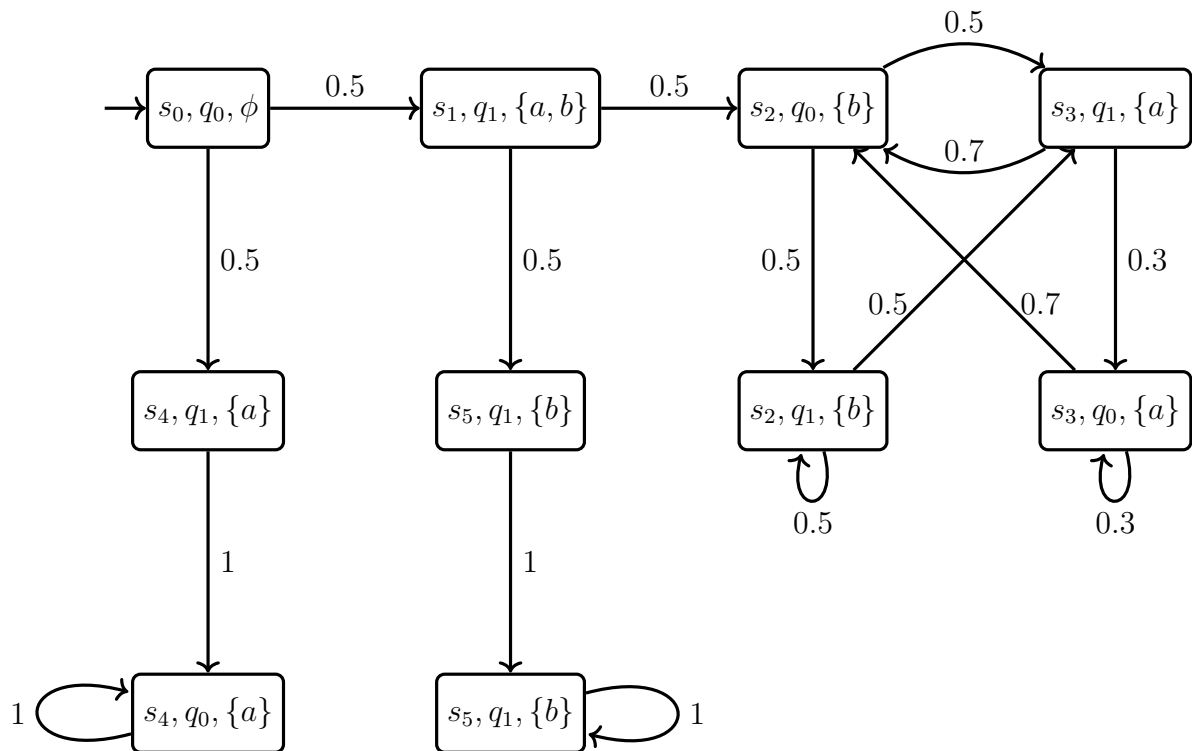


Acceptance:  $\{(\{q_0\}, \{q_1\}), (\{q_1\}, \{q_0\})\}$

We name all states in the chain for readability:

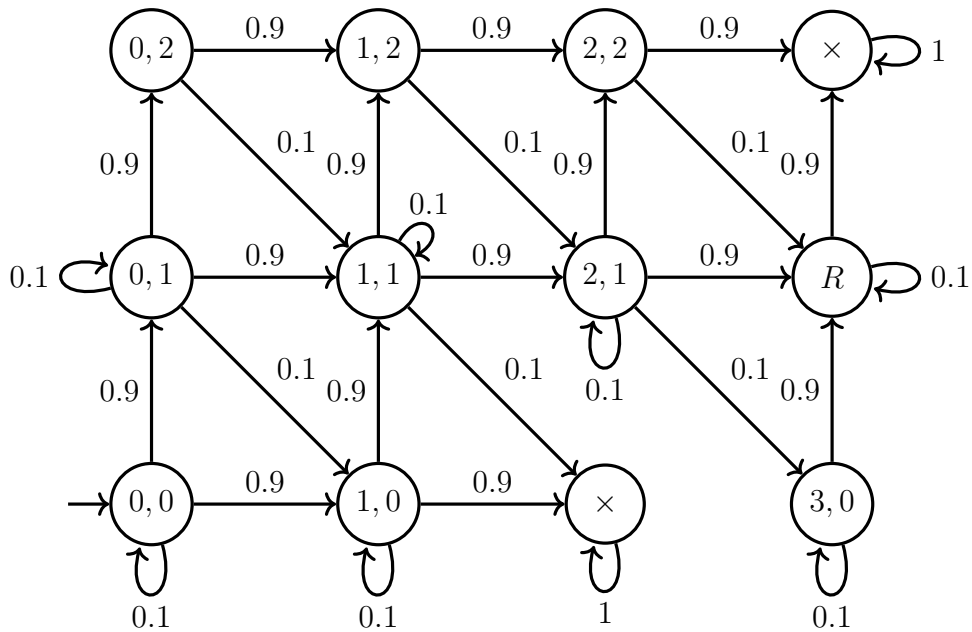


Now, we construct the Markov Chain-Automaton product, keeping the labels for readability.



In this product system, we search for all BSCCs and analyse them for acceptance. Only the  $\{(s_4, q_0)\}$  and  $\{(s_5, q_1)\}$  BSCCs are accepting, and this state set is reached with probability 0.75.

**Solution 9.3**



**Solution 9.4**

- $s_4$ : Prob = 1 with  $\{s_0 \mapsto a, s_1 \mapsto a\}$ .
- $\{s_0 \mapsto a, s_1 \mapsto b, s_3 \mapsto b, s_4 \mapsto b\}$ . There are uncountably many different strategies since we can randomize in, e.g.,  $s_2$ .