Technische Universität München (I7) **Winter Semester 2021-22**
J. Křetínský / A. Evangelidis / K. Grover **23.12.2021**

Quantitative Verification – Exercise sheet 8

For the questions below, we define the following notation $\mathbf{F}_{\sim p}\phi := \mathcal{P}_{\sim p}[\mathbf{F}\ \phi]$, and $\mathbf{G}_{\sim p}\phi := \mathcal{P}_{\sim p}[\mathbf{G}\ \phi]$ for $\sim\ \in \{=,<,\leq,>,\geq\}, p \in [0,1]$, and $\phi \in$ **PCTL**. We define analogous abbreviations for the step-bounded versions of **F** and **G** .

Exercise 8.1
Translate the following formulae to English

1. $\texttt{send} \Rightarrow \mathbf{F}^{\leq 10}_{\geq 0.95}\ \texttt{deliver}$
2. $\mathcal{P}_{\leq 0.05}\ \mathbf{F}\ \mathbf{F}_{\geq 0.9}\ \texttt{error}$
3. $\mathcal{P}_{\geq 0.8}\ \texttt{empty}\ \mathbf{U}\ (\texttt{send} \wedge \mathbf{G}_{\leq 0.5}\ \neg\texttt{receive})$

Exercise 8.2
Translate the following specifications into **PCTL/PLTL** formulae

1. The system with two processes satisfies mutual exclusion almost surely ($\texttt{crit}_i$ holds if process i is in the critical section).
2. The probability that every `request` will eventually be `grant`ed with a probability greater than 0.95 is 0.99.
3. The probability that component B fails (`B_fail`) before component A (`A_fail`) is less than 0.4.

Exercise 8.3
For each of the following properties, draw a labelled Markov Chain which satisfies it or argue why the property is unsatisfiable.

1. $\mathbf{G}_{\leq 0.5}\ (a \wedge \neg b)$
2. $\mathbf{G}_{=1}\ (\neg a \wedge \mathbf{F}_{=1}\ a)$
3. $\neg a \wedge \mathcal{P}_{=1}\ [b\ \mathbf{U}\ a]$
4. $\mathbf{F}_{=1}\ (a \Rightarrow (\mathbf{G}_{=1}\ ((b \Rightarrow c)\ \mathbf{U}\ \neg a \wedge (\mathbf{F}_{\geq 0.5}\ c \vee \neg b))))$

Solution 8.1

1. When there is a `send` in the first time step, with probability ≥ 0.95 there will be a deliver in the next 10 steps.

2. With probability ≤ 0.05, the system reaches a state which can reach `error` with probability ≥ 0.9.

3. With probability ≥ 0.8 the system is empty until it reaches a state with send and never receives with probability ≤ 0.5.

Solution 8.2

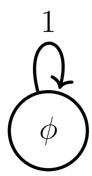
1. $\mathbf{G}_{=1}\ \neg(\texttt{crit}_1 \wedge \texttt{crit}_2)$

2. $\mathbf{G}_{=0.99}\ (\texttt{request} \Rightarrow \mathbf{F}_{\geq 0.95}\ \texttt{grant})$

3. $\mathcal{P}_{<0.4}\ [(\neg\texttt{A\_fail}\ \mathbf{U}\ (\texttt{B\_fail} \wedge \neg\texttt{A\_fail})) \vee \mathbf{G}\ \neg\texttt{B\_fail}]$

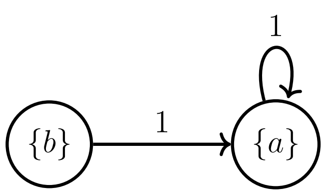
Solution 8.3

1. Consider the following automaton:



2. Not satisfiable. The formula requires to almost surely have a at every step and eventually reaching $\neg a$ with probability 1.

3. Consider the following automaton:



4. Look at the automaton from part 1 of this question.