

Quantitative Verification – Exercise sheet 8

For the questions below, we define the following notation $\mathbf{F}_{\sim p}\phi := \mathcal{P}_{\sim p}[\mathbf{F}\phi]$, and $\mathbf{G}_{\sim p}\phi := \mathcal{P}_{\sim p}[\mathbf{G}\phi]$ for $\sim \in \{=, <, \leq, >, \geq\}$, $p \in [0, 1]$, and $\phi \in \mathbf{PCTL}$. We define analogous abbreviations for the step-bounded versions of \mathbf{F} and \mathbf{G} .

Exercise 8.1

Translate the following formulae to English

1. $\text{send} \Rightarrow \mathbf{F}_{\geq 0.95}^{\leq 10} \text{deliver}$
2. $\mathcal{P}_{\leq 0.05} \mathbf{F} \mathbf{F}_{\geq 0.9} \text{error}$
3. $\mathcal{P}_{\geq 0.8} \text{empty} \mathbf{U} (\text{send} \wedge \mathbf{G}_{\leq 0.5} \neg \text{receive})$

Exercise 8.2

Translate the following specifications into $\mathbf{PCTL}/\mathbf{PLTL}$ formulae

1. The system with two processes satisfies mutual exclusion almost surely (crit_i holds if process i is in the critical section).
2. The probability that every **request** will eventually be **granted** with a probability greater than 0.95 is 0.99.
3. The probability that component B fails ($\mathbf{B_fail}$) before component A ($\mathbf{A_fail}$) is less than 0.4.

Exercise 8.3

For each of the following properties, draw a labelled Markov Chain which satisfies it or argue why the property is unsatisfiable.

1. $\mathbf{G}_{\leq 0.5} (a \wedge \neg b)$
2. $\mathbf{G}_{=1} (\neg a \wedge \mathbf{F}_{=1} a)$
3. $\neg a \wedge \mathcal{P}_{=1} [b \mathbf{U} a]$
4. $\mathbf{F}_{=1} (a \Rightarrow (\mathbf{G}_{=1} ((b \Rightarrow c) \mathbf{U} \neg a \wedge (\mathbf{F}_{\geq 0.5} c \vee \neg b))))$