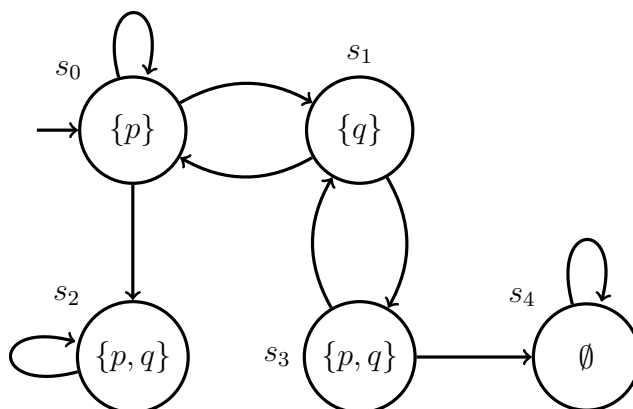


Model Checking – Exercise sheet 8

Exercise 8.1

Compute $\llbracket \mathbf{EG}q \rrbracket$, $\llbracket \mathbf{EXAG}(p \vee q) \rrbracket$ and $\llbracket \mathbf{EFAG}(p \wedge q) \rrbracket$ for the following Kripke structure:



Exercise 8.2

(Taken from *Principles of Model Checking*)

Provide two Kripke structures \mathcal{K}_1 and \mathcal{K}_2 (over the same set of atomic propositions) and a CTL formula ϕ such that $Traces(\mathcal{K}_1) = Traces(\mathcal{K}_2)$ and $\mathcal{K}_1 \models \phi$, but $\mathcal{K}_2 \not\models \phi$.

Exercise 8.3

Given two CTL formulas ϕ_1 and ϕ_2 , we write $\phi_1 \Rightarrow \phi_2$ iff for every Kripke structure \mathcal{K} we have $(\mathcal{K} \models \phi_1) \Rightarrow (\mathcal{K} \models \phi_2)$. Furthermore, we define an *implication graph* as a directed graph whose nodes are CTL formulas, and that contains an edge from ϕ_1 to ϕ_2 iff $\phi_1 \Rightarrow \phi_2$. Let $AP = \{p\}$.

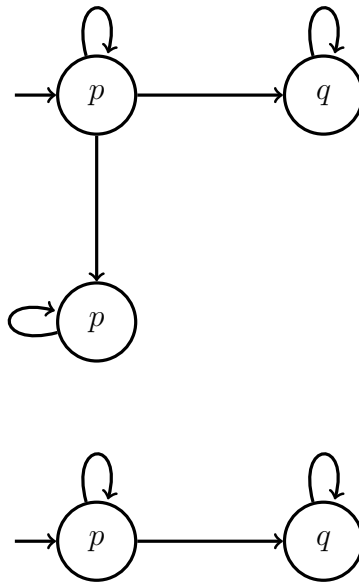
- (a) Draw an implication graph with the nodes: $\mathbf{EFEF}p$, $\mathbf{EGEG}p$, $\mathbf{AFAF}p$, $\mathbf{AGAG}p$.
- (b) For each implication $\phi_1 \Rightarrow \phi_2$ obtained in (a), give a Kripke structure \mathcal{K} that satisfies ϕ_2 but not ϕ_1 , i.e. give a \mathcal{K} such that $\mathcal{K} \models \phi_2$ and $\mathcal{K} \not\models \phi_1$.
- (c) Add the following CTL formulas to the implication graph obtained in (a): $\mathbf{AFEF}p$, $\mathbf{EFAF}p$, $\mathbf{AGEG}p$, $\mathbf{EGAG}p$.

Solution 8.1

1. $\llbracket \mathbf{EG}q \rrbracket = \{s_1, s_2, s_3\}$
2. $\llbracket \mathbf{EXAG}(p \vee q) \rrbracket = \{s_0\}$
3. $\llbracket \mathbf{EFAG}(p \wedge q) \rrbracket = \{s_0, s_1, s_2, s_3\}$

Solution 8.2

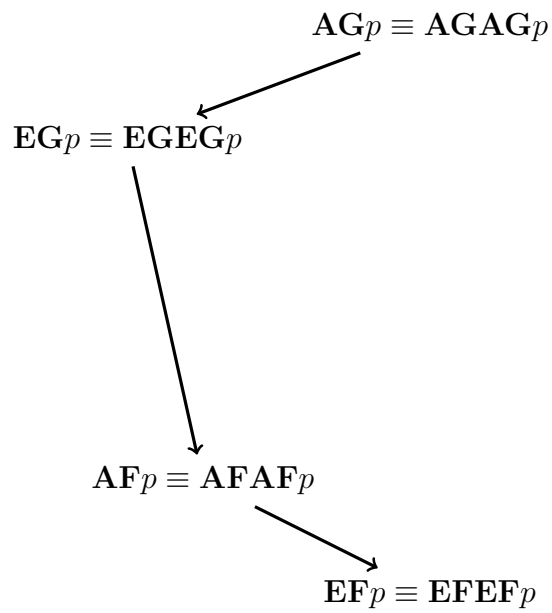
Consider the following Kripke structures and the property $\phi = \mathbf{EFAG}p$, first one satisfies ϕ and second one does not.



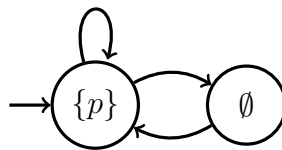
Solution 8.3

Note that the " \Rightarrow " relation is transitive, hence all transitive edges in (a), (b) and (d) are omitted.

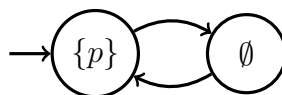
(a)



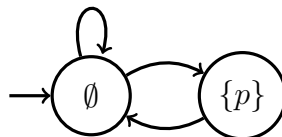
(b) The following Kripke structure satisfies $\mathbf{EG}p$, but not $\mathbf{AG}p$:



The following Kripke structure satisfies $\mathbf{AF}p$, but not $\mathbf{EG}p$:



The following Kripke structure satisfies $\mathbf{EF}p$, but not $\mathbf{AF}p$:



(c)

