Technische Universität München (I7) **Summer Semester 2022**
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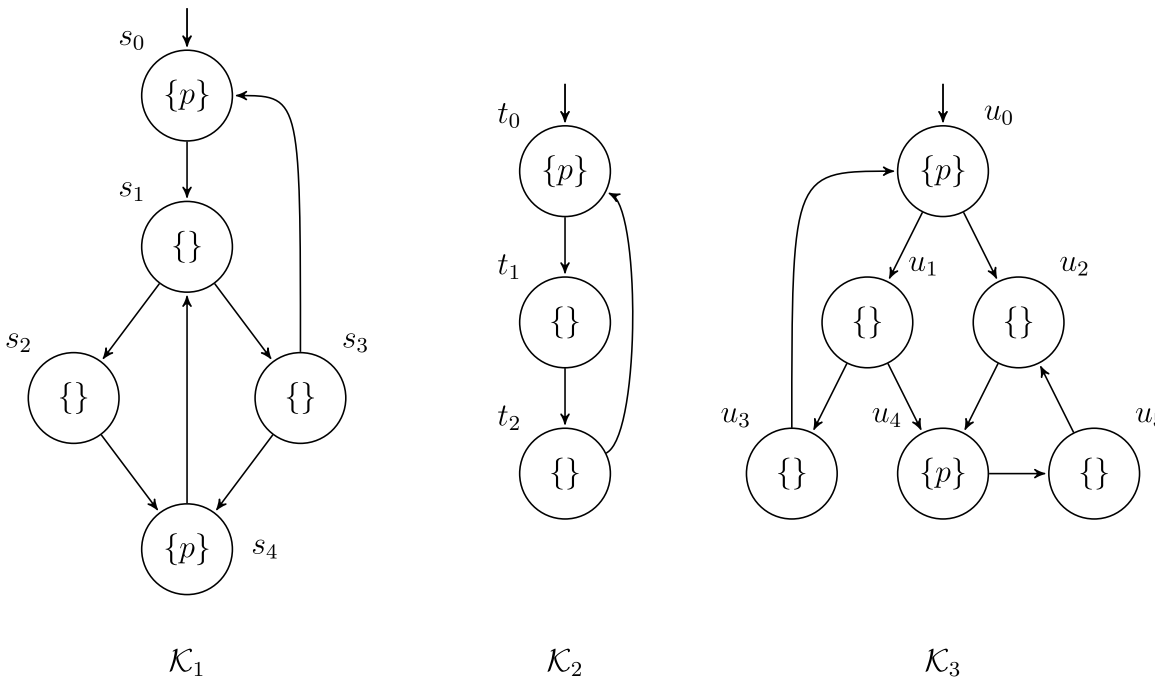
Model Checking – Exercise sheet 11

Exercise 11.1
Consider the following Kripke structures $\mathcal{K}_1$, $\mathcal{K}_2$, and $\mathcal{K}_3$, over $AP = \{p\}$:



(a) Does $\mathcal{K}_2$ simulate $\mathcal{K}_1$? If yes, give a simulation relation. Otherwise, explain why.

(b) Does $\mathcal{K}_2$ simulate $\mathcal{K}_3$? If yes, give a simulation relation. Otherwise, explain why.

(c) Does $\mathcal{K}_3$ simulate $\mathcal{K}_2$? If yes, give a simulation relation. Otherwise, explain why.

(d) Does $\mathcal{K}_3$ simulate $\mathcal{K}_1$? If yes, give a simulation relation. Otherwise, explain why.

Exercise 11.2
Let $\mathcal{K}_1$, $\mathcal{K}_2$, and $\mathcal{K}_3$ be Kripke structures. Show that if $\mathcal{K}_1$ and $\mathcal{K}_2$ are bisimilar, and $\mathcal{K}_2$ and $\mathcal{K}_3$ are bisimilar, then $\mathcal{K}_1$ and $\mathcal{K}_3$ are also bisimilar.

Solution 11.1

(a) Yes. $H = \{(s_0, t_0), (s_1, t_1), (s_2, t_2), (s_3, t_2), (s_4, t_0)\}$.

(b) No. If there exists a simulation H from $\mathcal{K}_3$ to $\mathcal{K}_2$, then we know that $(u_0, t_0) \in H$. Since $u_0 \to u_1$, we have $(u_1, t_1) \in H$. However, $u_1 \to u_4$ and u_4 satisfies p, but no successors of t_1 satisfy p, so H cannot exist.

(c) Yes. $H = \{(t_0, u_0), (t_1, u_1), (t_2, u_3\}$.

(d) Yes. $H = \{(s_0, u_0), (s_1, u_1), (s_2, u_3), (s_3, u_3), (s_4, u_0)\}$. Alternatively, we can also prove that $\mathcal{K}_1$ and $\mathcal{K}_2$ are bisimilar and use the result from (c).

Solution 11.2

Let H_{12} be a bisimulation between $\mathcal{K}_1$ and $\mathcal{K}_2$ and H_{23} be a bisimulation between $\mathcal{K}_2$ and $\mathcal{K}_3$. We define $H_{13} = \{(s, u) \mid \exists t : (s, t) \in H_{12} \wedge (t, u) \in H_{23}\}$ and show that H_{13} is a bisimulation between $\mathcal{K}_1$ and $\mathcal{K}_3$.

First, we prove that H_{13} is a simulation from $\mathcal{K}_1$ to $\mathcal{K}_3$. Basically, we need to prove that if $(s, u) \in H_{13}$ and $s \to_1 s'$, then there exists u' such that $u \to_3 u'$ and $(s', u') \in H_{13}$. From the definition of $(s, u) \in H_{13}$, we know that there exists t such that $(s, t) \in H_{12}$ and $(t, u) \in H_{23}$. Since $(s, t) \in H_{12}$ and $s \to_1 s'$, there must exist t' such that $t \to_2 t'$ and $(s', t') \in H_{12}$. Similarly, since $(t, u) \in H_{23}$ and $t \to_2 t'$, there must exist u' such that $u \to_3 u'$ and $(t', u') \in H_{23}$. Because $(s', t') \in H_{12}$ and $(t', u') \in H_{23}$, by the definition of H_{13} we have $(s', u') \in H_{13}$.

Analogously, we can prove that $\{(u, s) \mid (s, u) \in H_{13}\}$ is a simulation from $\mathcal{K}_3$ to $\mathcal{K}_1$.