Technische Universität München (I7) **Summer Semester 2022**
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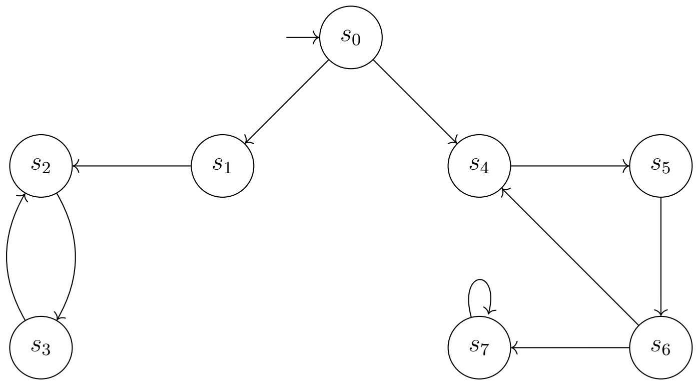
Model Checking – Exercise sheet 5

Exercise 5.1

Consider the following NBA with the acceptance set $F = \{s_1, s_6\}$. Apply the nested depth-first search approach to verify that $L(A) \neq \phi$.



Exercise 5.2

Consider the Promela model below which addresses the mutual exclusion problem by using a semaphore `s`. When `s` is false, a process may enter its critical section and set `s` to true. The semaphore is reset to false when the process leaves its critical section.

```
bool s;

active [2] proctype m() {
        idle:
        skip;
        wait:
        atomic { (!s) -> s = true; }
        cs:
        s = false;
        goto idle;
}
```

We consider the following properties:

a) Both processes cannot enter the critical section at the same time.

b) Whenever a process waits, it will eventually enter the critical section.

Follow step-by-step the outline given below to model check the properties:

(i) Construct a state transition system from the model.

(ii) Write down an atomic proposition AP and an LTL formula ϕ for each properties.

(iii) Construct a Büchi automaton $\mathcal{B}_{\neg\phi}$ for the negation of the formula ϕ.

(iv) Construct from the transition system the Kripke structure $\mathcal{K}$ and the Büchi automaton $\mathcal{B}_{\mathcal{K}}$ over AP.

(v) Construct the intersection Büchi automaton $\mathcal{B}$ for $\mathcal{B}_{\mathcal{K}}$ and $\mathcal{B}_{\neg\phi}$.

(vi) Run the emptiness algorithm in the lecture to check whether $\mathcal{L}(\mathcal{B}) = \emptyset$:

- If $\mathcal{L}(\mathcal{B}) = \emptyset$, the property holds, i.e. $\mathcal{K} \models \phi$.
- If $\mathcal{L}(\mathcal{B}) \neq \emptyset$, the property does not hold, i.e. $\mathcal{K} \not\models \phi$.
  In this case, find a counterexample run that violates the property. How to obtain a counterexample in general?

(vii) Use Spin to confirm your results.

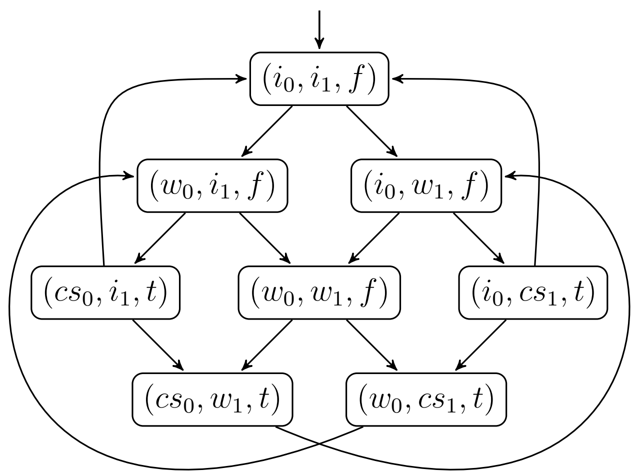
First do step (i), and then steps (ii)-(vii) separately for each property (a) and (b). Write down all intermediary results.

Solution 5.1

Let's assume that the algorithm selects the next state from top-to-bottom and left-to-right. It first finds the SCC $\{s_2, s_3\}$ which does not have any accepting state. Then it goes to state s_4 and later finds the SCC $\{s_4, s_5, s_6\}$ when it reaches s_6. Since s_6 is an accepting state, the algorithm terminates and reports that the language of the NBA is non-empty.

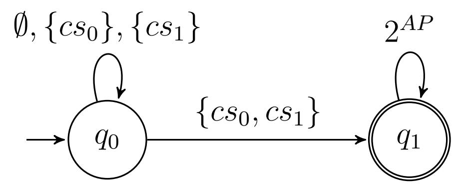
Solution 5.2

(i) $\mathcal{T} = (S, \rightarrow, r)$, where $S = \{i_0, w_0, cs_0\} \times \{i_1, w_1, cs_1\} \times \{t, f\}$ for modeling three locations in `m` and two possible values of `s`.

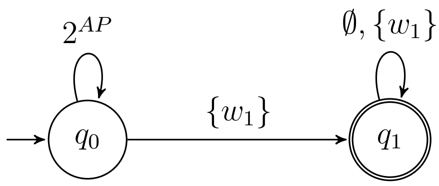


(ii) $\phi_a = \neg\mathbf{F}(cs_0 \wedge cs_1)$, where $AP_a = \{cs_0, cs_1\}$ and
$\phi_b = \mathbf{G}(w_0 \rightarrow \mathbf{F}cs_0)$, where $AP_b = \{w_0, cs_0\}$

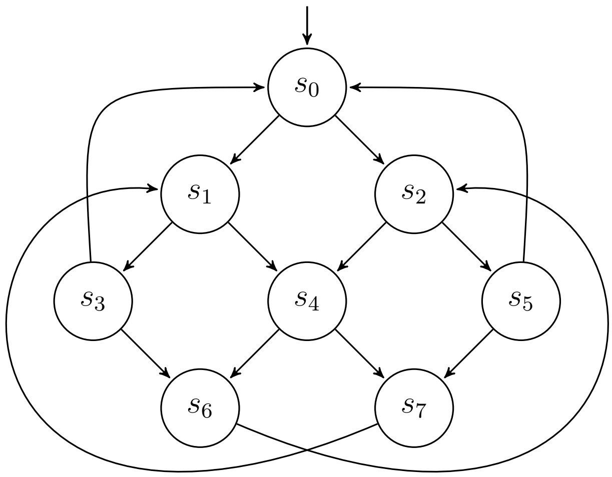
(iii) $\neg\phi_a = \mathbf{F}(cs_1 \wedge cs_2)$. So, $\mathcal{B}_{\neg\phi_a}$ can be constructed as follows:



$\neg\phi_b = \mathbf{F}(w_0 \wedge \mathbf{G}\neg cs_0)$. So, $\mathcal{B}_{\neg\phi_b}$ can be constructed as follows:



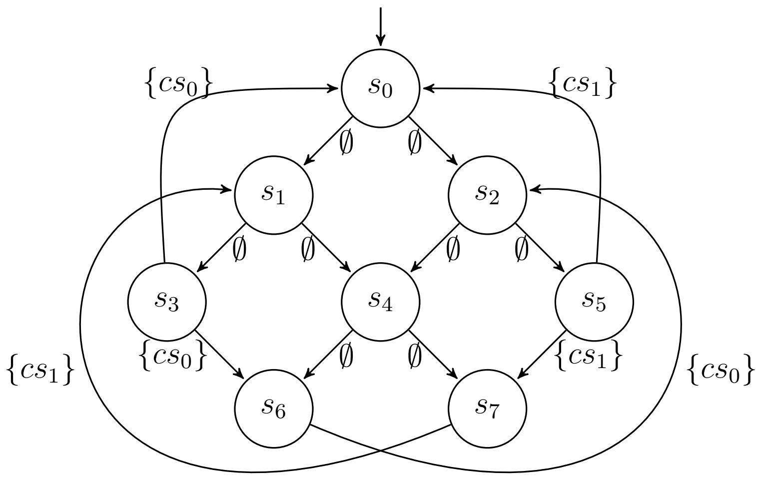
(iv) Let rename the states to $S = \{s_0, \dots, s_7\}$.



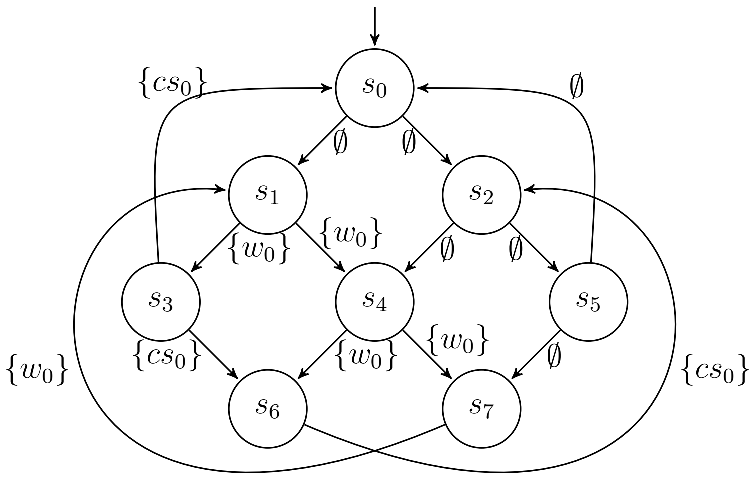
$$\mathcal{K}_a = (S, \to, r, AP_a, \nu_a), \text{ where } \nu_a(s) = \begin{cases} \{cs_0\}, & \text{if } s \in \{s_3, s_6\} \\ \{cs_1\}, & \text{if } s \in \{s_5, s_7\} \\ \emptyset, & \text{otherwise} \end{cases}$$

$$\mathcal{K}_b = (S, \to, r, AP_b, \nu_b), \text{ where } \nu_b(s) = \begin{cases} \{w_0\}, & \text{if } s \in \{s_1, s_4, s_7\} \\ \{cs_0\}, & \text{if } s \in \{s_3, s_6\} \\ \emptyset, & \text{otherwise} \end{cases}$$

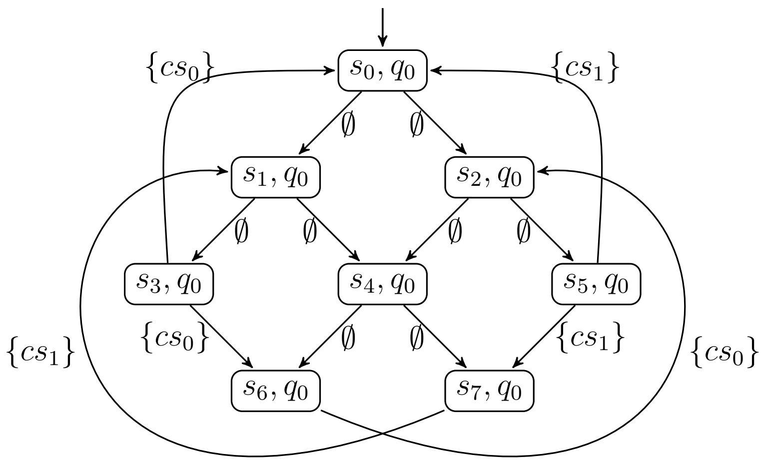
The Büchi automaton $\mathcal{B}_{\mathcal{K}_a} = (2^{AP_a}, S, r, \Delta_{K_a}, S)$, where $\Delta_{K_a} = \{(s, \nu_a(s), t) \mid s \to t\}$:



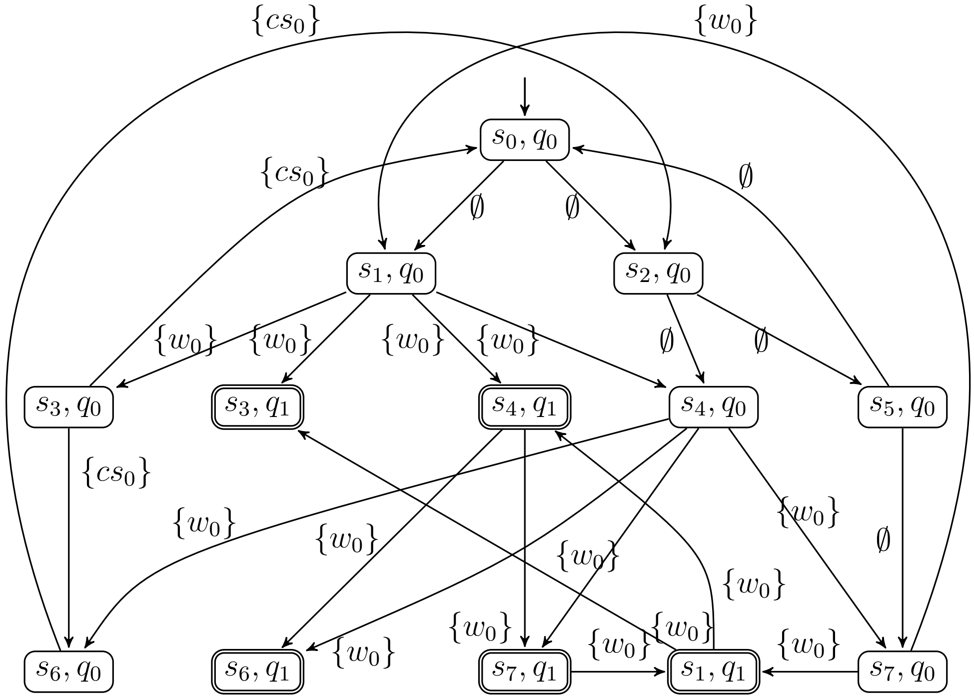
The Büchi automaton $\mathcal{B}_{\mathcal{K}_b} = (2^{AP_b}, S, r, \Delta_{K_b}, S)$, where $\Delta_{K_b} = \{(s, \nu_b(s), t) \mid s \to t\}$:



(v) $\mathcal{B}_a = (2^{AP_a}, S \times \{q_0, q_1\}, (s_0, q_0), \Delta_a, S \times \{q_1\})$



$\mathcal{B}_b = (2^{AP_b}, S \times \{q_0, q_1\}, (s_0, q_0), \Delta_b, S \times \{q_1\})$



(vi) For $\mathcal{B}_a$, the algorithm terminates without reporting a counterexample. By the time the algorithm terminates, it finds out that every state forms an SCC, but without an accepting state.

For $\mathcal{B}_b$, assuming that the algorithm always searches the automaton above top-to-bottom and left-to-right, then it first finds the SCC $\{(s_0, q_0), (s_1, q_0), (s_3, q_0)\}$ when it reaches (s_3, q_0). Later, it finds the SCC $\{(s_6, q_0), (s_2, q_0), (s_4, q_0)\}$ when it reaches (s_4, q_0). Finally, it reaches (s_4, q_1) and finds out that its successor (s_7, q_1) is still active. Since (s_4, q_1) is an accepting state, the algorithm stops. Notice that the last SCC it discovers is $\{(s_7, q_1), (s_1, q_1), (s_4, q_1)\}$ with (s_7, q_1) as the root.

The counterexample run in $\mathcal{B}_b$ found by the algorithm is

$$(s_0, q_0), (s_1, q_0), (s_3, q_0), (s_6, q_0), (s_2, q_0), (s_4, q_0), ((s_7, q_1), (s_1, q_1), (s_4, q_1))^\omega$$

The corresponding path in $\mathcal{K}_b$ can be obtained by projecting the run on the first component. The counterexample valuation sequence is as follows:

$$\{\}\{w_0\}\{cs_0\}\{cs_0\}\{\}\{w_0\}(\{w_0\}\{w_0\}\{w_0\})^\omega$$

(vii) Check the formulas: `!<>(m[0]@cs && m[1]@cs)` and `[](m@wait -> (<> m@cs))`