Model Checking – Exercise sheet 4

Exercise 4.1

Using the *Compare* feature in Spot (`https://spot.lrde.epita.fr/app`) give an LTL formula equivalent to

(a) $p \, \mathbf{R} \, q$, which does not contain $\neg$ but may contain $\mathbf{U}, \mathbf{G}$ or $\mathbf{F}$.

(b) $(\mathbf{G}p) \, \mathbf{U} \, q$ which does not contain $\mathbf{U}$.

(c) $(\mathbf{F}p) \, \mathbf{U} \, q$, which does not contain $\mathbf{U}$.

Exercise 4.2

Given the following Kripke structures and LTL formulae, answer the following questions

(a) Which of $\mathcal{K}_1, \mathcal{K}_2$ and $\mathcal{K}_3$ satisfy $\phi = \mathbf{G}(\mathbf{X}q \to p)$?

(b) Give an LTL formula which exactly characterizes $\mathcal{K}_3$, i.e. both the formula and the Kripke structure accept exactly the same words.
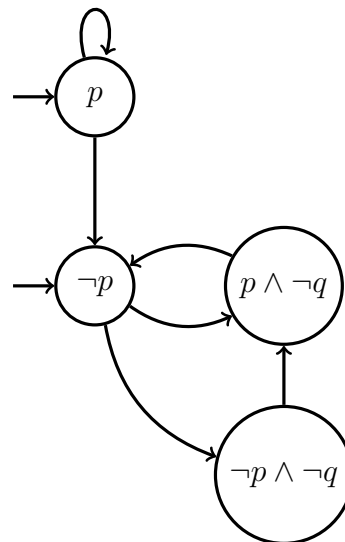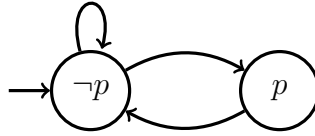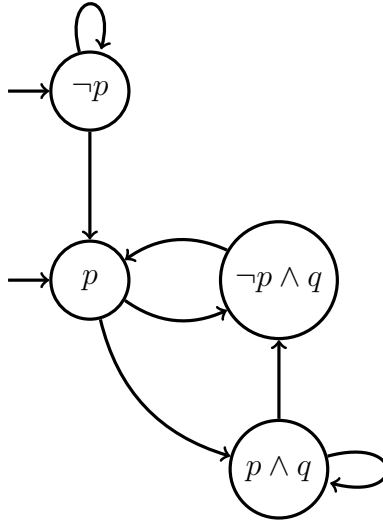

Figure 1: $\mathcal{K}_1$
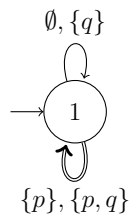
Figure 2: $\mathcal{K}_2$
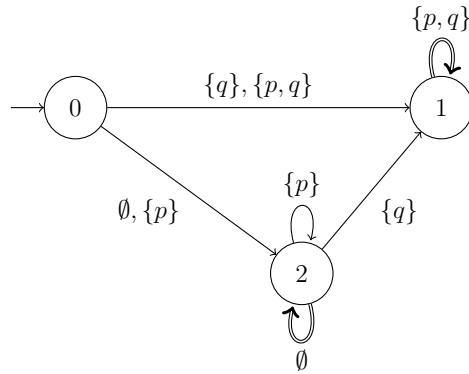

Figure 3: $\mathcal{K}_3$

Exercise 4.3

Convert the following Büchi automata with transition-based acceptance condition ("doubled"-transitions have to be seen infinitely often) to equivalent Büchi automata with state-based acceptance conditions. Moreover, give a general procedure to perform this conversion.

(a)

(b)

Exercise 4.4

Extend the set of rules of the LTL to Büchi automata translation to directly deal with the **F** and **G** operators.

Exercise 4.5

Let $\phi = \mathbf{G}\left((\mathbf{X}(p\ \mathbf{U}\ q)) \rightarrow ((\neg p \wedge \mathbf{F}q) \vee (q\ \mathbf{U}\ \mathbf{X}q))\right)$ and $\mathcal{G}$ be a generalized Büchi automaton translated from ϕ using the construction presented in the lecture and the extended set of rules defined in the previous exercise.

(a) Write down the set of subformulae $Sub(\phi)$.

(b) What is the size of $CS(\phi)$?

(c) How many sets of accepting states does $\mathcal{G}$ have?

(d) Is $\{\phi\}$ an accepting state of $\mathcal{G}$?

(e) Give a reachable state that has no successors.

(f) Give a successor state of the smallest consistent state containing $\{\phi, q, q\ \mathbf{U}\ \mathbf{X}q, \mathbf{F}q\}$.

(g) Give a predecessor state of the smallest consistent state containing $\{\phi, q, q\ \mathbf{U}\ \mathbf{X}q, \mathbf{F}q\}$.