

Exercise Sheet 9

Assignment 9.1 Type Checking

This exercise is about checking the types of expressions given in our C-like language. Use a deduction tree to check whether the statements are well-typed. Make sure to only use the rules given in the lecture and specify the rule for each step.

1. Given the declarations $\Gamma := \{\text{int } x, \text{int } a[]\}$, check whether the statement $\text{int } y = x + a[42]$; is well-typed.
2. Given the declarations $\Gamma := \{\text{int } y, \text{double } a[], \text{struct } \{\text{double } a[]; \} g, \text{int } (*f)(\text{double})\}$, check whether the statement $\text{int } x = f(g.a[y + 2])$; is well-typed.

Suggested Solution 9.1

1.

$$\text{OP} \frac{\text{VAR} \frac{}{\Gamma \vdash x : \text{int}} \quad \text{ARRAY} \frac{\text{VAR} \frac{}{\Gamma \vdash a : \text{int}[]} \quad \text{CONST} \frac{}{\Gamma \vdash 42 : \text{int}}}{\Gamma \vdash a[42] : \text{int}}}{\Gamma \vdash x + a[42] : \text{int}}$$

2.

$$\begin{array}{c}
\text{VAR } \frac{\Gamma \vdash g : \text{struct}\{\text{double}[] a;\}}{\Gamma \vdash g.a : \text{double}[]} \\
\text{STRUCT} \frac{}{\Gamma \vdash g.a : \text{double}[]} \\
\text{ARRAY} \frac{}{\Gamma \vdash g.a[y+2] : \text{double}} \\
\text{OP} \frac{\text{VAR } \frac{\Gamma \vdash y : \text{int}}{\Gamma \vdash y + 2 : \text{int}} \quad \text{CONST } \frac{}{\Gamma \vdash 2 : \text{int}}}{\Gamma \vdash y + 2 : \text{int}} \\
\text{VAR} \frac{\Gamma \vdash f : \text{int}^*(\text{double}) \quad \Gamma \vdash g.a[y+2] : \text{double}}{\Gamma \vdash f(g.a[y+2]) : ?}
\end{array}$$

There is no rule in our type system that can be applied to `f : int(*) (double)` and `g.a[y+2] : double`.

Assignment 9.2 Subtyping

Consider the following C structs:

```
struct A {
    A f(B, C);
    C g(C);
}
```

```
struct B {
    B f(A, D);
    A g(D);
}
```

```
struct C {
    C f(B, B);
    D g(A);
}
```

```
struct D {
    D f(B, B);
    D g(B);
    int a;
}
```

We are going to use the non-standard subtyping rules for C structures which have been introduced in the lecture. Let \leq be the type comparison operator, that is, for two types A and B the following holds:

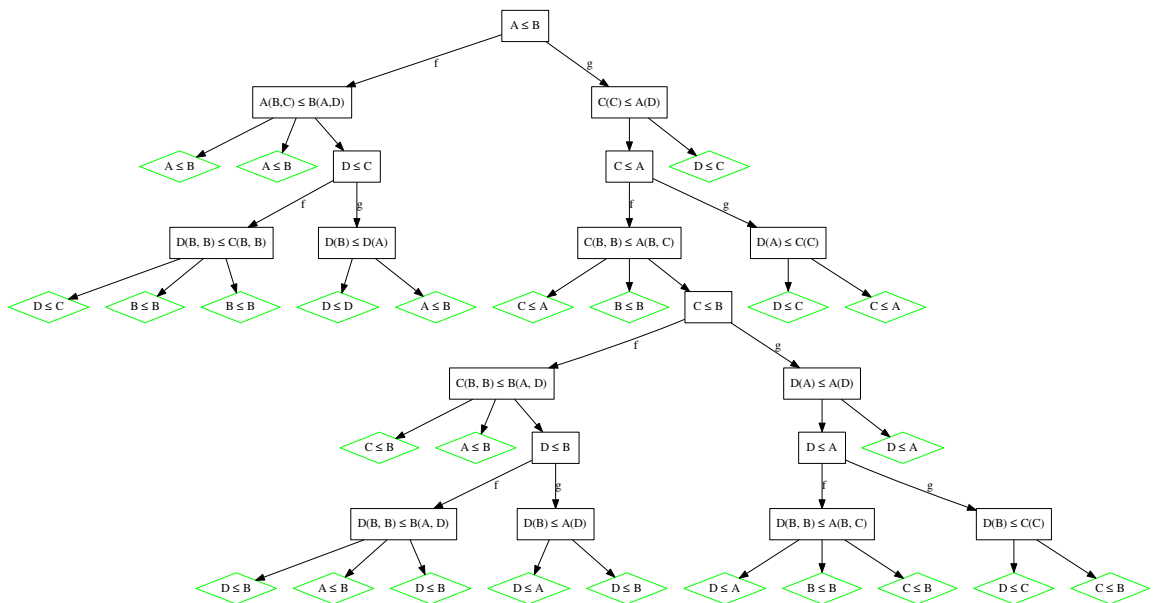
$$A \leq B \Leftrightarrow A \text{ is a subtype of } B \quad (1)$$

Now, prove the assertions below either right or wrong:

1. $A \leq B$
2. $A \leq C$

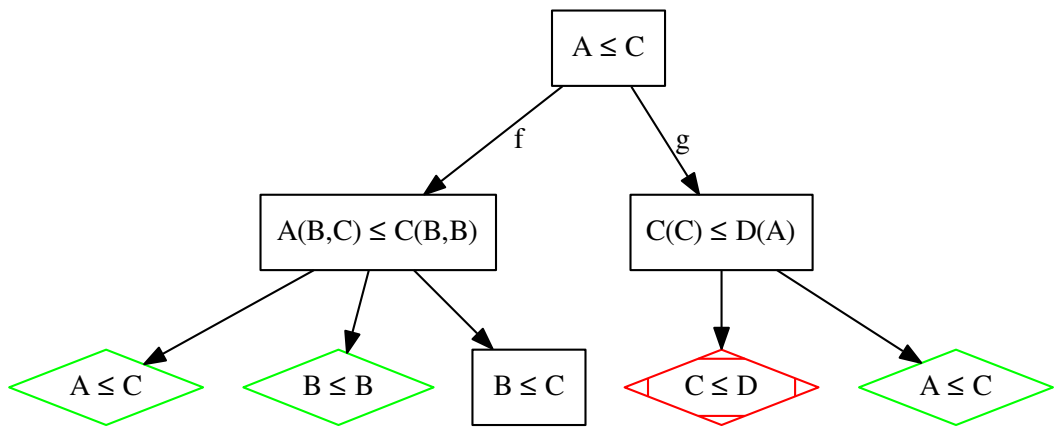
Suggested Solution 9.2

1.



Since no contradictions can be found, it follows that $A \leq B$ holds.

2.



Since D contains a field $(D.a)$ that is not contained in C , $C \leq D$ cannot hold. Therefore, $A \leq C$ does not hold.