

Topic:

Code Synthesis

Generating Code: Overview

We inductively generate instructions from the AST:

- there is a rule stating how to generate code for each non-terminal of the grammar
- the code is merely another attribute in the syntax tree
- code generation makes use of the already computed attributes

In order to specify the code generation, we require

- a semantics of the language we are compiling (here: C standard)
- a semantics of the machine instructions

~> we commence by specifying machine instruction semantics

Chapter 1: The Register C-Machine

The Register C-Machine (R-CMa)

We generate Code for the Register C-Machine.

The Register C-Machine is a virtual machine (VM).

- there exists no processor that can execute its instructions
- . . . but we can build an interpreter for it
- we provide a visualization environment for the R-CMa
- the R-CMa has no **double**, **float**, **char**, **short** or **long** types
- the R-CMa has no instructions to communicate with the operating system
- the R-CMa has an unlimited supply of registers

The R-CMa is more realistic than it may seem:

- the mentioned restrictions can easily be lifted
- the *Dalvik VM/ART* or the *LLVM* are similar to the R-CMa
- an interpreter of R-CMa can run on any platform

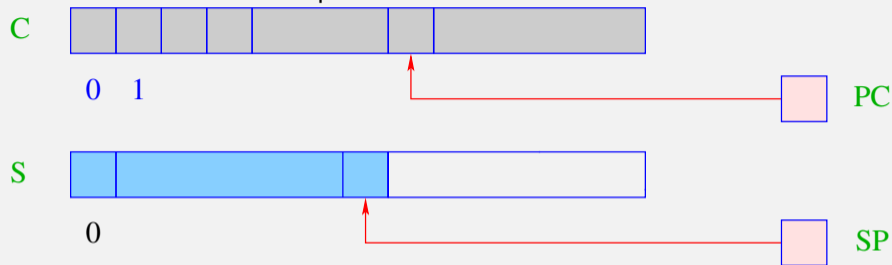
Virtual Machines

A virtual machine has the following ingredients:

- any virtual machine provides a set of **instructions**
- instructions are executed on virtual hardware
- the virtual hardware is a collection of data structures that is accessed and modified by the VM instructions
- ... and also by other components of the **run-time system**, namely functions that go beyond the instruction semantics
- the **interpreter** is part of the run-time system

Components of a Virtual Machine

Consider **Java** as an example:



A virtual machine such as the **Dalvik VM** has the following structure:

- **S**: the data store – a memory region in which cells can be stored in LIFO order \rightsquigarrow stack.
- **SP**: ($\hat{=}$ stack pointer) pointer to the last used cell in **S**
- beyond **S** follows the memory containing the heap
- **C** is the memory storing **code**
 - each cell of **C** holds exactly one virtual instruction
 - **C** can only be **read**
- **PC** ($\hat{=}$ program counter) address of the instruction that is to be executed next
- **PC** contains 0 initially

Executing a Program

- the machine loads an instruction from `C[PC]` into the `instruction register IR` in order to execute it
- before evaluating the instruction, the `PC` is incremented by one

```
while (true) {  
    IR = C[PC]; PC++;  
    execute (IR);  
}
```

- note: the `PC` must be incremented `before` the execution, since an instruction may modify the `PC`
- the loop is exited by evaluating a `halt` instruction that returns directly to the operating system

Chapter 2: Generating Code for the Register C-Machine

Simple Expressions and Assignments in R-CMa

Task: evaluate the expression $(1 + 7) * 3$
that is, generate an instruction sequence that

- computes the value of the expression and
- keeps its value accessible in a reproducible way

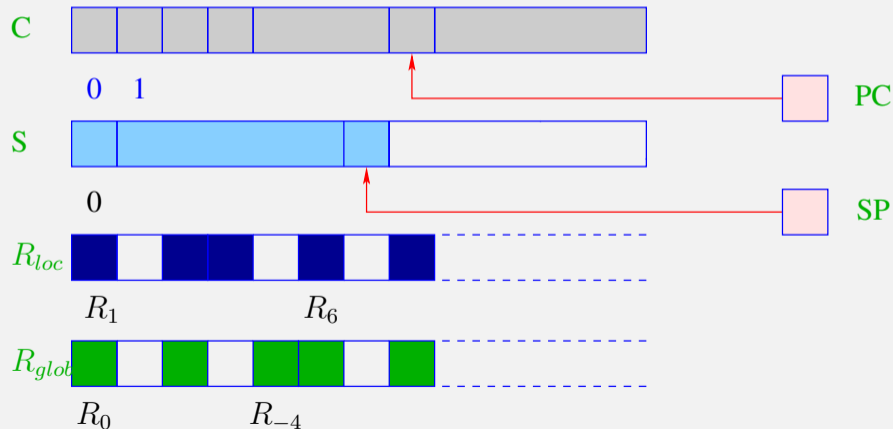
Idea:

- first compute the value of the sub-expressions
- store the intermediate result in a temporary register
- apply the operator
- loop

Principles of the R-CMa

The **R-CMa** is composed of a stack, heap and a code segment, just like the **JVM**; it additionally has register sets:

- **local** registers are $R_1, R_2, \dots, R_i, \dots$
- **global** register are $R_0, R_{-1}, \dots, R_j, \dots$



The Register Sets of the R-CMa

The two register sets have the following purpose:

- 1 the *local* registers R_i
 - save temporary results
 - store the contents of local variables of a function
 - can efficiently be stored and restored from the stack
- 2 the *global* registers R_i
 - save the parameters of a function
 - store the result of a function

Note:

for now, we only use registers to store temporary computations

Idea for the translation: use a register counter i :

- registers R_j with $j < i$ are *in use*
- registers R_j with $j \geq i$ are *available*

Translation of Simple Expressions

Using variables stored in registers; loading constants:

instruction	semantics	intuition
<code>loadc</code> R_i c	$R_i = c$	load constant
<code>move</code> R_i R_j	$R_i = R_j$	copy R_j to R_i

We define the following translation schema (with $\rho x = a$):

$$\begin{aligned}\text{code}_{\mathbf{R}}^i c \rho &= \text{loadc } R_i c \\ \text{code}_{\mathbf{R}}^i x \rho &= \text{move } R_i R_a \\ \text{code}_{\mathbf{R}}^i x = e \rho &= \text{code}_{\mathbf{R}}^i e \rho \\ &\quad \text{move } R_a R_i\end{aligned}$$

Translation of Expressions

Let $op = \{add, sub, div, mul, mod, le, gr, eq, leq, geq, and, or\}$. The R-CMa provides an instruction for each operator op .

$$op \ R_i \ R_j \ R_k$$

where R_i is the target register, R_j the first and R_k the second argument.

Correspondingly, we generate code as follows:

$$\begin{aligned} \text{code}_{\mathbf{R}}^i \ e_1 \ op \ e_2 \ \rho &= \text{code}_{\mathbf{R}}^i \ e_1 \ \rho \\ &\quad \text{code}_{\mathbf{R}}^{i+1} \ e_2 \ \rho \\ &\quad \text{op} \ R_i \ R_i \ R_{i+1} \end{aligned}$$

Example: Translate $3 * 4$ with $i = 4$:

$$\begin{aligned} \text{code}_{\mathbf{R}}^4 \ 3 * 4 \ \rho &= \text{code}_{\mathbf{R}}^4 \ 3 \ \rho \\ &\quad \text{code}_{\mathbf{R}}^5 \ 4 \ \rho \\ \text{code}_{\mathbf{R}}^4 \ 3 * 4 \ \rho &= \text{loadc} \ R_4 \ 3 \\ &\quad \text{loadc} \ R_5 \ 4 \\ &\quad \text{mul} \ R_4 \ R_4 \ R_5 \end{aligned}$$

Managing Temporary Registers

Observe that temporary registers are re-used: translate $3 * 4 + 3 * 4$ with $t = 4$:

$$\text{code}_{\mathbf{R}}^4 \ 3 * 4 + 3 * 4 \ \rho = \begin{array}{l} \text{code}_{\mathbf{R}}^4 \ 3 * 4 \ \rho \\ \text{code}_{\mathbf{R}}^5 \ 3 * 4 \ \rho \\ \text{add } R_4 \ R_4 \ R_5 \end{array}$$

where

$$\text{code}_{\mathbf{R}}^i \ 3 * 4 \ \rho = \begin{array}{l} \text{loadc } R_i \ 3 \\ \text{loadc } R_{i+1} \ 4 \\ \text{mul } R_i \ R_i \ R_{i+1} \end{array}$$

we obtain

$$\text{code}_{\mathbf{R}}^4 \ 3 * 4 + 3 * 4 \ \rho = \begin{array}{l} \text{loadc } R_4 \ 3 \\ \text{loadc } R_5 \ 4 \\ \text{mul } R_4 \ R_4 \ R_5 \\ \text{loadc } R_5 \ 3 \\ \text{loadc } R_6 \ 4 \\ \text{mul } R_5 \ R_5 \ R_6 \\ \text{add } R_4 \ R_4 \ R_5 \end{array}$$

Semantics of Operators

The operators have the following semantics:

add $R_i R_j R_k$	$R_i = R_j + R_k$
sub $R_i R_j R_k$	$R_i = R_j - R_k$
div $R_i R_j R_k$	$R_i = R_j / R_k$
mul $R_i R_j R_k$	$R_i = R_j * R_k$
mod $R_i R_j R_k$	$R_i = \text{signum}(R_k) \cdot k$ with $ R_j = n \cdot R_k + k \wedge n \geq 0, 0 \leq k < R_k $
le $R_i R_j R_k$	$R_i = \text{if } R_j < R_k \text{ then } 1 \text{ else } 0$
gr $R_i R_j R_k$	$R_i = \text{if } R_j > R_k \text{ then } 1 \text{ else } 0$
eq $R_i R_j R_k$	$R_i = \text{if } R_j = R_k \text{ then } 1 \text{ else } 0$
leq $R_i R_j R_k$	$R_i = \text{if } R_j \leq R_k \text{ then } 1 \text{ else } 0$
geq $R_i R_j R_k$	$R_i = \text{if } R_j \geq R_k \text{ then } 1 \text{ else } 0$
and $R_i R_j R_k$	$R_i = R_j \& R_k$ // bit-wise and
or $R_i R_j R_k$	$R_i = R_j R_k$ // bit-wise or

Note: all registers and memory cells contain operands in \mathbb{Z}

Translation of Unary Operators

Unary operators $\text{op} = \{\text{neg}, \text{not}\}$ take only two registers:

$$\text{code}_{\text{R}}^i \text{ op } e \ \rho \quad = \quad \text{code}_{\text{R}}^i e \ \rho \\ \text{op } R_i \ R_i$$

Note: We use the same register.

Example: Translate -4 into R_5 :

$$\text{code}_{\text{R}}^5 -4 \ \rho \quad = \quad \text{code}_{\text{R}}^5 4 \ \rho \\ \text{code}_{\text{R}}^5 -4 \ \rho \quad = \quad \text{loadc } R_5 \ 4 \\ \text{neg } R_5 \ R_5$$

The operators have the following semantics:

$$\begin{array}{ll} \text{not } R_i \ R_j & R_i \leftarrow \text{if } R_j = 0 \text{ then } 1 \text{ else } 0 \\ \text{neg } R_i \ R_j & R_i \leftarrow -R_j \end{array}$$

Applying Translation Schema for Expressions

Suppose the following function
is given:

```
void f(void) {  
    int x, y, z;  
    x = y+z*3;  
}
```

- Let $\rho = \{x \mapsto 1, y \mapsto 2, z \mapsto 3\}$ be the address environment.
- Let R_4 be the first free register, that is, $i = 4$.

$$\text{code}_R^4 \ x=y+z*3 \ \rho = \text{code}_R^4 \ y+z*3 \ \rho$$

move $R_1 \ R_4$

$$\text{code}_R^4 \ y+z*3 \ \rho = \text{move } R_4 \ R_2$$

$\text{code}_R^5 \ z*3 \ \rho$
add $R_4 \ R_4 \ R_5$

$$\text{code}_R^5 \ z*3 \ \rho = \text{move } R_5 \ R_3$$

$\text{code}_R^6 \ 3 \ \rho$
mul $R_5 \ R_5 \ R_6$

$$\text{code}_R^6 \ 3 \ \rho = \text{loadc } R_6 \ 3$$

\leadsto the assignment $x=y+z*3$ is translated as

move $R_4 \ R_2$; move $R_5 \ R_3$; loadc $R_6 \ 3$; mul $R_5 \ R_5 \ R_6$; add $R_4 \ R_4 \ R_5$; move $R_1 \ R_4$

Chapter 3: Statements and Control Structures

About Statements and Expressions

General idea for translation: $\text{code}^i s \rho$: generate code for statement s
 $\text{code}_R^i e \rho$: generate code for expression e into R_i

Throughout: $i, i + 1, \dots$ are free (unused) registers

For an *expression* $x = e$ with $\rho x = a$ we defined:

$$\text{code}_R^i x = e \rho = \text{code}_R^i e \rho \\ \text{move } R_a R_i$$

However, $x = e$; is also an *expression statement*:

- Define:

$$\text{code}^i e_1 = e_2; \rho = \text{code}_R^i e_1 = e_2 \rho$$

The temporary register R_i is ignored here. More general:

$$\text{code}^i e; \rho = \text{code}_R^i e \rho$$

- **Observation:** the assignment to e_1 is a side effect of the evaluating the expression $e_1 = e_2$.

Translation of Statement Sequences

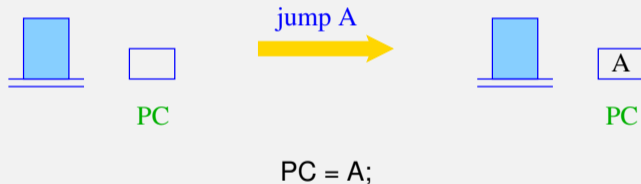
The code for a sequence of statements is the concatenation of the instructions for each statement in that sequence:

$$\begin{aligned} \text{code}^i (s \text{ } ss) \rho &= \text{code}^i s \rho \\ &\quad \text{code}^i ss \rho \\ \text{code}^i \varepsilon \rho &= // \textit{empty sequence of instructions} \end{aligned}$$

Note here: s is a statement, ss is a sequence of statements

Jumps

In order to diverge from the linear sequence of execution, we need *jumps*:



Conditional Jumps

A conditional jump branches depending on the value in R_i :



if ($R_i == 0$) PC = A;

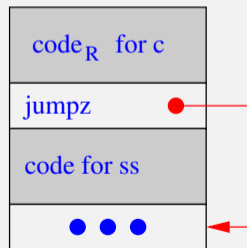
Simple Conditional

We first consider $s \equiv \mathbf{if} (c) ss$.

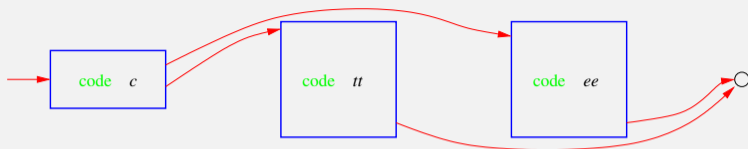
...and present a translation without basic blocks.

Idea:

- emit the code of c and ss in sequence
- insert a jump instruction in-between, so that correct control flow is ensured

$$\begin{aligned} \text{code}^i s \rho &= \text{code}_R^i c \rho \\ &\quad \text{jumpz } R_i A \\ &\quad \text{code}^i ss \rho \\ A : &\quad \dots \end{aligned}$$


General Conditional



Translation of **if** (*c*) *tt* **else** *ee*.

$\text{code}^i \text{ if}(c) \text{ } tt \text{ else } ee \rho =$

$\text{code}_R^i c \rho$

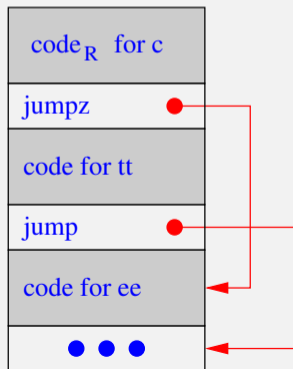
$\text{jumpz } R_i A$

$\text{code}^i tt \rho$

$\text{jump } B$

$A : \text{code}^i ee \rho$

$B :$



Example for if-statement

Let $\rho = \{x \mapsto 4, y \mapsto 7\}$ and let s be the statement

```
if (x>y) { /* (i) */  
    x = x - y; /* (ii) */  
} else {  
    y = y - x; /* (iii) */  
}
```

Then $\text{code}^i s \rho$ yields:

(i)

```
move  $R_i R_4$   
move  $R_{i+1} R_7$   
gr  $R_i R_i R_{i+1}$   
jumpz  $R_i A$ 
```

(ii)

```
move  $R_i R_4$   
move  $R_{i+1} R_7$   
sub  $R_i R_i R_{i+1}$   
move  $R_4 R_i$   
jump  $B$ 
```

(iii)

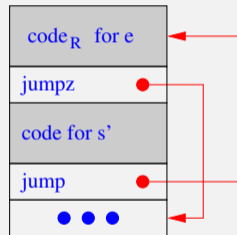
```
A : move  $R_i R_7$   
     move  $R_{i+1} R_4$   
     sub  $R_i R_i R_{i+1}$   
     move  $R_7 R_i$   
B :
```

Iterating Statements

We only consider the loop $s \equiv \mathbf{while} (e) s'$. For this statement we define:

$code^i \mathbf{while}(e) s \rho = A :$ $code^i_R e \rho$
 $jumpz R_i B$
 $code^i s \rho$
 $jump A$

$B :$



Example: Translation of Loops

Let $\rho = \{a \mapsto 7, b \mapsto 8, c \mapsto 9\}$ and let s be the statement:

```
while (a>0) {      /* (i) */
    c = c + 1;      /* (ii) */
    a = a - b;      /* (iii) */
}
```

Then $\text{code}^i s \rho$ evaluates to:

(i)	(ii)	(iii)
A : <code>move R_i R_7</code>	<code>move R_i R_9</code>	<code>move R_i R_7</code>
<code>loadc R_{i+1} 0</code>	<code>loadc R_{i+1} 1</code>	<code>move R_{i+1} R_8</code>
<code>gr R_i R_i R_{i+1}</code>	<code>add R_i R_i R_{i+1}</code>	<code>sub R_i R_i R_{i+1}</code>
<code>jumpz R_i B</code>	<code>move R_9 R_i</code>	<code>move R_7 R_i</code>
		<code>jump A</code>

B :

for-Loops

The **for**-loop $s \equiv \mathbf{for} (e_1; e_2; e_3) s'$ is equivalent to the statement sequence $e_1; \mathbf{while} (e_2) \{s' e_3; \}$ – as long as s' does not contain a **continue** statement.

Thus, we translate:

$$\begin{aligned} \text{code}^i \mathbf{for}(e_1; e_2; e_3) s \rho &= \text{code}_R^i e_1 \rho \\ A : \text{code}_R^i e_2 \rho & \\ \text{jumpz } R_i B & \\ \text{code}^i s \rho & \\ \text{code}_R^i e_3 \rho & \\ \text{jump } A & \\ B : & \end{aligned}$$

The switch-Statement

Idea:

- Suppose choosing from multiple options in *constant time* if possible
- use a *jump table* that, at the i th position, holds a jump to the i th alternative
- in order to realize this idea, we need an *indirect jump* instruction



$$PC = A + R_i;$$

Consecutive Alternatives

Let **switch** s be given with k consecutive **case** alternatives:

```

switch (e) {
  case 0:  $s_0$ ; break;
  :
  case  $k - 1$ :  $s_{k-1}$ ; break;
  default:  $s_k$ ; break;
}

```

Define $\text{code}^i s \rho$ as follows:

$\text{code}^i s \rho$	=	$\text{code}_R^i e \rho$		$B :$	$\text{jump } A_0$
			$\text{check}^i 0 k B$		\vdots
$A_0 :$		$\text{code}^i s_0 \rho$			\vdots
		$\text{jump } C$			$\text{jump } A_k$
		\vdots		$C :$	
		\vdots			
$A_k :$		$\text{code}^i s_k \rho$			
		$\text{jump } C$			

$\text{check}^i l u B$ checks if $l \leq R_i < u$ holds and jumps accordingly.

Translation of the *checkⁱ* Macro

The macro *checkⁱ* $l\ u\ B$ checks if $l \leq R_i < u$. Let $k = u - l$.

- if $l \leq R_i < u$ it jumps to $B + R_i - l$
- if $R_i < l$ or $R_i \geq u$ it jumps to A_k

we define:

```
checki  $l\ u\ B$  =   loadc  $R_{i+1}\ l$   
                    geq  $R_{i+2}\ R_i\ R_{i+1}$     $B :$    jump  $A_0$   
                    jumpz  $R_{i+2}\ E$   
                    sub  $R_i\ R_i\ R_{i+1}$         $:$         $:$   
                    loadc  $R_{i+1}\ k$   
                    geq  $R_{i+2}\ R_i\ R_{i+1}$        jump  $A_k$   
                    jumpz  $R_{i+2}\ D$             $C :$   
 $E :$    loadc  $R_i\ k$   
 $D :$    jumpi  $R_i\ B$ 
```

Note: a jump `jumpi $R_i\ B$` with $R_i = u$ winds up at $B + u$, the default case

Improvements for Jump Tables

This translation is only suitable for *certain* **switch**-statement.

- In case the table starts with 0 instead of u we don't need to subtract it from e before we use it as index
- if the value of e is **guaranteed** to be in the interval $[l, u]$, we can omit *check*

General translation of switch-Statements

In general, the values of the various cases may be far apart:

- generate an **if**-ladder, that is, a sequence of **if**-statements
- for n cases, an **if**-cascade (tree of conditionals) can be generated $\sim O(\log n)$ tests
- if the sequence of numbers has small gaps (≤ 3), a jump table may be smaller and faster
- one could generate several jump tables, one for each sets of consecutive cases
- an **if** cascade can be re-arranged by using information from *profiling*, so that paths executed more frequently require fewer tests

Chapter 4: Functions

Ingredients of a Function

The definition of a function consists of

- a **name** with which it can be called;
- a specification of its **formal parameters**;
- possibly a **result type**;
- a sequence of **statements**.

In C we have:

$\text{code}_R^i f \rho = \text{loadc } R_i _f$ with $_f$ starting address of f

Observe:

- function names must have an address assigned to them
- since the size of functions is unknown before they are translated, the addresses of forward-declared functions must be inserted later

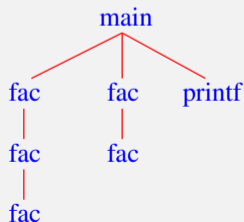
Memory Management in Functions

```
int fac(int x) {  
    if (x<=0) return 1;  
    else return x*fac(x-1);  
}
```

```
int main(void) {  
    int n;  
    n = fac(2) + fac(1);  
    printf("%d", n);  
}
```

At run-time several **instances** may be active, that is, the function has been called but has not yet returned.

The recursion tree in the example:



Memory Management in Function Variables

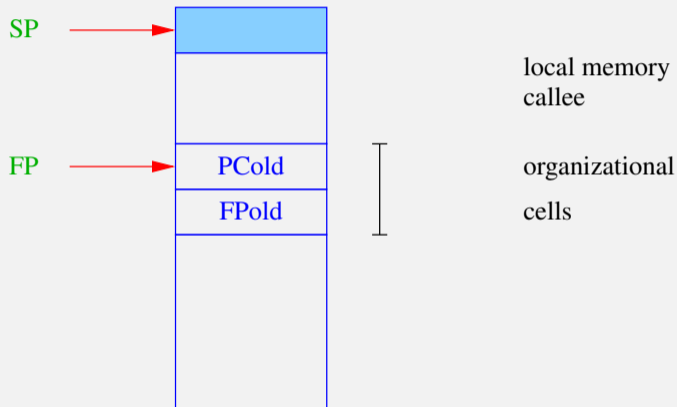
The **formal parameters** and the **local variables** of the various **instances** of a function must be kept separate

Idea for implementing functions:

- set up a region of memory each time it is called
- in sequential programs this memory region can be allocated on the stack
- thus, each instance of a function has its own region on the stack
- these regions are called **stack frames**

Organization of a Stack Frame

- **stack** representation: grows upwards
- **SP** points to the last used stack cell



- **FP** $\hat{=}$ **frame pointer**: points to the last **organizational cell**
- used to recover the previously active stack frame

Split of Obligations

Definition

Let f be the current function that calls a function g .

- f is dubbed *caller*
- g is dubbed *callee*

The code for managing function calls has to be split between caller and callee. This split cannot be done arbitrarily since some information is only known in that caller or only in the callee.

Observation:

The space requirement for parameters is only known by the caller:

Example: `printf`

Principle of Function Call and Return

actions taken on **entering** g :

1. compute the start address of g
2. compute actual parameters in globals
3. backup of **caller-save** registers
4. backup of **FP**
5. set the new **FP**
6. back up of **PC** and
jump to the beginning of g
7. copy actual params to locals

} **saveloc**
} **mark** } are in f
} **call**
} ... } is in g

actions taken on **leaving** g :

1. compute the result into R_0
2. restore **FP**, **SP**
3. return to the call site in f ,
that is, restore **PC**
4. restore the **caller-save** registers

} **return** } are in g
} **restoreloc** } is in f

Managing Registers during Function Calls

The two register sets (global and local) are used as follows:

- automatic variables live in *local* registers R_i
- intermediate results also live in *local* registers R_i
- parameters live in *global* registers R_i (with $i \leq 0$)
- global variables: let's suppose there are none

convention:

- the i th argument of a function is passed in register R_{-i}
- the result of a function is stored in R_0
- local registers are saved before calling a function

Definition

Let f be a function that calls g . A register R_i is called

- *caller-saved* if f backs up R_i and g may overwrite it
- *callee-saved* if f does not back up R_i , and g must restore it before returning

Translation of Function Calls

A function call $g(e_1, \dots, e_n)$ is translated as follows:

$$\begin{aligned} \text{code}_R^i g(e_1, \dots, e_n) \rho &= \text{code}_R^i g \rho \\ &\quad \text{code}_R^{i+1} e_1 \rho \\ &\quad \vdots \\ &\quad \text{code}_R^{i+n} e_n \rho \\ &\quad \text{move } R_{-1} R_{i+1} \\ &\quad \vdots \\ &\quad \text{move } R_{-n} R_{i+n} \\ &\quad \text{saveloc } R_1 R_{i-1} \\ &\quad \text{mark} \\ &\quad \text{call } R_i \\ &\quad \text{restoreloc } R_1 R_{i-1} \\ &\quad \text{move } R_i R_0 \end{aligned}$$

New instructions:

- **saveloc** $R_i R_j$ pushes the registers $R_i, R_{i+1} \dots R_j$ onto the stack
- **mark** backs up the organizational cells
- **call** R_i calls the function at the address in R_i
- **restoreloc** $R_i R_j$ pops R_j, R_{j-1}, \dots, R_i off the stack

Rescuing the FP

The instruction `mark` allocates stack space for the return value and the organizational cells and backs up `FP`.



```
S[SP+1] = FP;  
SP = SP + 1;
```

Calling a Function

The instruction `call Ri` rescues the value of `PC+1` onto the stack and sets `FP` and `PC`.



```
SP = SP+1;  
S[SP] = PC;  
FP = SP;  
PC = Ri;
```

Result of a Function

The global register set is also used to communicate the result value of a function:

$$\text{code}^i \text{ return } e \rho = \begin{array}{l} \text{code}_R^i e \rho \\ \text{move } R_0 R_i \\ \text{return} \end{array}$$

alternative without result value:

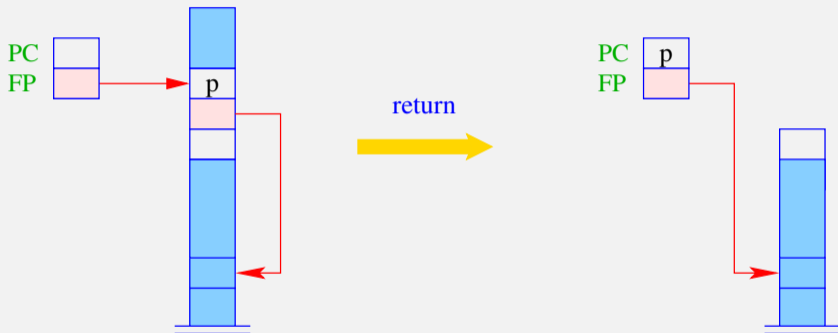
$$\text{code}^i \text{ return } \rho = \text{return}$$

global registers are otherwise not used inside a function body:

- advantage: at any point in the body another function can be called without backing up *global* registers
- disadvantage: on entering a function, all *global* registers must be saved

Return from a Function

The instruction `return` relinquishes control of the current stack frame, that is, it restores `PC` and `FP`.



```
PC = S[FP];  
SP = FP-2;  
FP = S[SP+1];
```

Translation of Functions

The translation of a function is thus defined as follows:

$$\begin{aligned} \text{code}^1 \text{ } t_r \text{ } f(\text{args})\{\text{decls } ss\} \rho &= \text{move } R_{l+1} \text{ } R_{-1} \\ &\quad \vdots \\ &\quad \text{move } R_{l+n} \text{ } R_{-n} \\ &\quad \text{code}^{l+n+1} \text{ } ss \text{ } \rho' \\ &\quad \text{return} \end{aligned}$$

Assumptions:

- the function has n parameters
- the local variables are stored in registers R_1, \dots, R_l
- the parameters of the function are in R_{-1}, \dots, R_{-n}
- ρ' is obtained by extending ρ with the bindings in decls and the function parameters args
- **return** is not always necessary

Are the **move** instructions always necessary?

Translation of Whole Programs

A program $P = F_1; \dots F_n$ must have a single `main` function.

```
code1 P ρ    =   loadc R1 _main
                  mark
                  call R1
                  halt
    _f1 : code1 F1 ρ ⊕ ρf1
                ⋮
    _fn : code1 Fn ρ ⊕ ρfn
```

Assumptions:

- $\rho = \emptyset$ assuming that we have no global variables
- ρ_{f_i} contain the addresses of the functions up to f_i
- $\rho_1 \oplus \rho_2 = \lambda x . \begin{cases} \rho_2(x) & \text{if } x \in \text{dom}(\rho_2) \\ \rho_1(x) & \text{otherwise} \end{cases}$

Translation of the `fac`-function

Consider:

```
int fac(int x) {  
    if (x<=0)  
        return 1;  
    else  
        return x*fac(x-1);  
}
```

```
_fac:  move R1 R-1  save param.  
i = 2  move R2 R1   if (x<=0)  
      loadc R3 0  
      leq R2 R2 R3  
      jumpz R2 _A  to else  
      loadc R2 1  return 1  
      move R0 R2  
      return  
      jump _B    code is dead
```

```
_A:  move R2 R1      x*fac(x-1)  
i = 3  loadc R3 _fac  
i = 4  move R4 R1    x-1  
i = 5  loadc R5 1  
i = 6  sub R4 R4 R5  
i = 5  move R-1 R4  fac(x-1)  
i = 3  saveloc R1 R2  
      mark  
      call R3  
      restoreloc R1 R2  
      move R3 R0  
i = 4  mul R2 R2 R3  
i = 3  move R0 R2    return x*...  
_B:  return
```