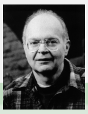
Topic:

Syntactic Analysis - Part II

Chapter 1:
Bottom-up Analysis

Shift-Reduce Parser


Donald Knuth

Idea:
We *delay* the decision whether to reduce until we know, whether the input matches the right-hand-side of a rule!

Construction: Shift-Reduce parser M_G^R

- The input is shifted successively to the pushdown.
- Is there a complete right-hand side (a handle) atop the pushdown, it is replaced (reduced) by the corresponding left-hand side

Shift-Reduce Parser

Example:

$$\begin{array}{lll} S & \to & A\,B \\ A & \to & a \\ B & \to & b \end{array}$$

The pushdown automaton:

States: q_0, f, a, b, A, B, S;
Start state: q_0
End state: f

q_0	a	$q_0\,a$
a	ϵ	A
A	b	$A\,b$
b	ϵ	B
$A\,B$	ϵ	S
$q_0\,S$	ϵ	f

Shift-Reduce Parser

Construction:
In general, we create an automaton $M_G^R = (Q, T, \delta, q_0, F)$ with:

- $Q = T \cup N \cup \{q_0, f\}$ (q_0, f fresh);
- $F = \{f\}$;
- Transitions:

$$\begin{array}{rcll} \delta & = & \{(q, x, q\,x) \mid q \in Q, x \in T\} \cup & // \text{ Shift-transitions} \\ & & \{(\alpha, \epsilon, A) \mid A \to \alpha \in P\} \cup & // \text{ Reduce-transitions} \\ & & \{(q_0\,S, \epsilon, f)\} & // \text{ finish} \end{array}$$

Example-computation:

$$\begin{array}{rcrcr} (q_0, \; a\,b) & \vdash & (q_0\;a\;, \; b) & \vdash & (q_0\;A, \; b) \\ & \vdash & (q_0\;A\;b\;, \; \epsilon) & \vdash & (q_0\;A\,B\;, \; \epsilon) \\ & \vdash & (q_0\,S, \; \epsilon) & \vdash & (f, \; \epsilon) \end{array}$$

Shift-Reduce Parser

Observation:

- The sequence of reductions corresponds to a reverse rightmost-derivation for the input
- To prove correctnes, we have to prove:

$$(\epsilon, w) \vdash^* (A, \epsilon) \quad \text{iff} \quad A \to^* w$$

- The shift-reduce pushdown automaton M_G^R is in general also non-deterministic
- For a deterministic parsing-algorithm, we have to identify computation-states for reduction

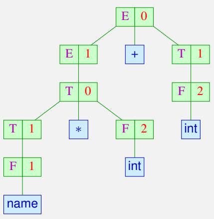
$\Longrightarrow$ LR-Parsing

The Pushdown During an RR-Derivation

Idea: Observe a successful run of M_G^R!

Input:
counter $* \; 2 + 40$

Pushdown:
(q_0)



$$\begin{array}{lllll} E & \to & E{+}T\;^0 & \mid & T\;^1 \\ T & \to & T{*}F\;^0 & \mid & F\;^1 \\ F & \to & (\;E\;)\;^0 & \mid & \text{name}\;^1 \quad \mid \quad \text{int}\;^2 \end{array}$$

Result:

Viable Prefixes and Admissable Items

Formalism: use *Items* as representations of *prefixes of righthandsides*

Generic Agreement
In a sequence of configurations of M_G^R

$$(q_0\,\alpha\,\gamma, \; v) \vdash (q_0\,\alpha\,B, \; v) \vdash^* (q_0\,S, \; \epsilon)$$

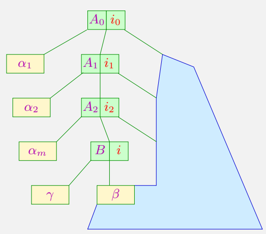
we call $\alpha\,\gamma$ a viable prefix for the complete item $[B \to \gamma\bullet]$.

Reformulating the Shift-Reduce-Parsers main problem:
Find the items, for which the content of M_G^R's stack is the viable prefix....

$\to$ *Admissable Items*

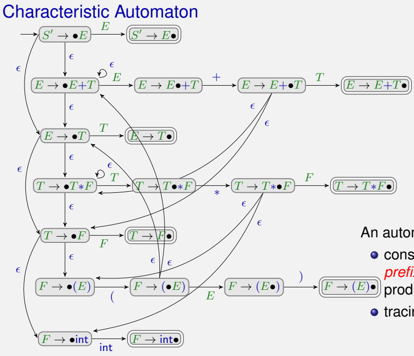
Admissible Items

The item $[B \to \gamma \bullet \beta]$ is called admissible for $\alpha\gamma$ iff $S \to_R^* \alpha\,B\,v$:



... with $\alpha = \alpha_1 \ldots \alpha_m$

Characteristic Automaton



An automaton...

- consuming pushdown symbols, i.e. *prefixes of righthandsides* of productions expanding from S
- tracing admissible items in its states

Characteristic Automaton

Observation:
One can now consume the shift-reduce parser's pushdown with the characteristic automaton: If the input $(N\cup T)^*$ for the characteristic automaton corresponds to a viable prefix, its state contains the admissible items.

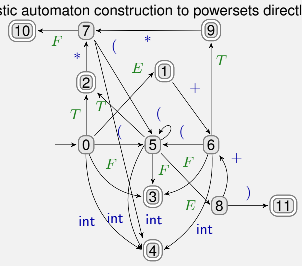
States: Items
Start state: $[S'\to\bullet\, S]$
Final states: $\{[B\to\gamma\bullet]\mid B\to\gamma\in P\}$
Transitions:
(1) $([A\to\alpha\bullet X\,\beta],X,[A\to\alpha\,X\bullet\beta])$, $\quad X\in(N\cup T),\ A\to\alpha\,X\,\beta\in P$;
(2) $([A\to\alpha\bullet B\,\beta],\epsilon,\ [B\to\bullet\,\gamma])$, $\quad A\to\alpha\,B\,\beta\ ,\ \ B\to\gamma\in P$;

The automaton $c(G)$ is called characteristic automaton for G.

Canonical LR(0)-Automaton

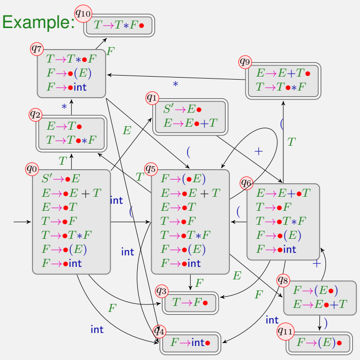
The canonical $LR(0)$-automaton $LR(G)$ is created from $c(G)$ by:
- performing arbitrarily many ϵ-transitions after every consuming transition
- performing the powerset construction
- Idea: or rather apply characteristic automaton construction to powersets directly?

... for example:



Canonical LR(0)-Automaton – Example:

$$
\begin{array}{lcl}
S' & \to & E \\
E & \to & E+T \ \mid\ T \\
T & \to & T*F \ \mid\ F \\
F & \to & (\,E\,) \ \mid\ \text{int}
\end{array}
$$



Canonical LR(0)-Automaton

Observation:

The canonical $LR(0)$-automaton can be created directly from the grammar.
For this we need a helper function δ^*_ϵ (ϵ-closure)

$$
\delta^*_\epsilon(q) = q \cup \{[B\to\bullet\,\gamma] \mid B\to\gamma\in P,\ [A\to\alpha\bullet B'\,\beta']\in q,\ B'\to^* B\,\beta\}
$$

We define:

States: Sets of items;
Start state: $\delta^*_\epsilon\,\{[S'\to\bullet\,S]\}$
Final states: $\{q\mid [A\to\alpha\,\bullet]\in q\}$
Transitions: $\delta(q,X) = \delta^*_\epsilon\,\{[A\to\alpha\,X\bullet\beta]\mid [A\to\alpha\bullet X\,\beta]\in q\}$

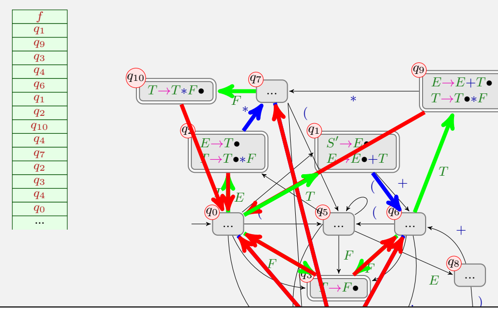
LR(0)-Parser

Idea for a parser:
- The parser manages a viable prefix $\alpha = X_1\ldots X_m$ on the pushdown and uses $LR(G)$ to identify reduction spots.
- It can reduce with $A\to\gamma$, if $[A\to\gamma\,\bullet]$ is admissible for α

Optimization:
We push the states instead of the X_i in order not to process the pushdown's content with the automaton anew all the time.
Reduction with $A\to\gamma$ leads to popping the uppermost $|\gamma|$ states and continue with the state on top of the stack and input A.

Attention:
This parser is only deterministic, if each final state of the canonical $LR(0)$-automaton is conflict free.

LR(0)-Parser – Example:



LR(0)-Parser

... we observe:

q_1: $S'\to E\bullet$, $E\to E\bullet+T$

q_2: $E\to T\bullet$, $T\to T\bullet*F$

q_3: $T\to F\bullet$

q_4: $F\to\text{int}\bullet$

q_9: $E\to E+T\bullet$, $T\to T\bullet*F$

q_{10}: $T\to T*F\bullet$

q_{11}: $F\to(E)\bullet$

The final states q_1, q_2, q_9 contain more than one admissible item

$\Rightarrow$ non-deterministic!

LR(0)-Parser

The construction of the $LR(0)$-parser:

States: $Q\cup\{f\}$ (f fresh)
Start state: q_0
Final state: f
Transitions:

Shift:	$(p,a,p\,q)$	if	$q=\delta(p,a)\neq\emptyset$
Reduce:	$(p\,q_1\ldots q_m,\epsilon,p\,q)$	if	$[A\to X_1\ldots X_m\,\bullet]\in q_m,\quad q=\delta(p,A)$
Finish:	$(q_0\,p,\epsilon,f)$	if	$[S'\to S\bullet]\in p$

with the canonical automaton $LR(G) = (Q,T,\delta,q_0,F)$.

LR(0)-Parser

Correctness:

we show:

The accepting computations of an $LR(0)$-parser are one-to-one related to those of a shift-reduce parser M^R_G.

we conclude:
- The accepted language is exactly $\mathcal{L}(G)$
- The sequence of reductions of an accepting computation for a word $w\in T$ yields a reverse rightmost derivation of G for w

LR(0)-Parser

Attention:
Unfortunately, the $LR(0)$-parser is in general non-deterministic.

We identify two reasons for a state $q\in Q$:

Reduce-Reduce-Conflict:

q: $A\to\gamma\,\bullet$, $A'\to\gamma'\,\bullet$ with $A\neq A'\vee\gamma\neq\gamma'$

Shift-Reduce-Conflict:

q: $A\to\gamma\,\bullet$, $A'\to\alpha\bullet a\,\beta$ with $a\in T$

Those states are called $LR(0)$-unsuited.

Revisiting the Conflicts of the LR(0)-Automaton

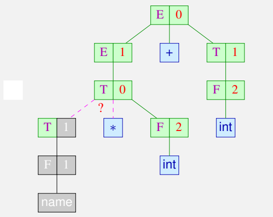
What differenciates the particular Reductions and Shifts?

Input:

$* \, 2 + 40$

Pushdown:

$(\, q_0 \, T \,)$



$$E \;\to\; E + T \;\mid\; T$$
$$T \;\to\; T * F \;\mid\; F$$
$$F \;\to\; (\,E\,) \;\mid\; \text{int}$$

LR(k)-Grammars

Idea: Consider k-lookahead in conflict situations.

Definition:

The reduced contextfree grammar G is called $LR(k)$-grammar, if $\alpha\,\beta\,w|_{|\alpha\beta|+k} = \alpha'\,\beta'\,w'|_{|\alpha\beta|+k}$ with:

$$\left.\begin{array}{lllll} S & \to_R^* & \alpha\,A\,w & \to & \alpha\,\beta\,w \\ S & \to_R^* & \alpha'\,A'\,w' & \to & \alpha'\,\beta'\,w' \end{array}\right\} \text{ follows: } \alpha = \alpha' \;\wedge\; \beta = \beta' \;\wedge\; A = A'$$

Strategy for testing Grammars for $LR(k)$-property

- Focus iteratively on all rightmost derivations $S \to_R^* \alpha\,X\,w \to \alpha\,\beta\,w$
- Iterate over $k \geq 0$
  - For each $\gamma = \alpha\,\beta w|_{|\alpha\beta|+k}$ (*handle with k-lookahead*) check if there exists a differently right-derivable $\alpha'\,\beta' w'$ for which $\gamma = \alpha'\,\beta' w'|_{|\alpha\beta|+k}$
  - if there is none, we have found no objection against k being enough lookahead to disambiguate $\alpha\beta w$ from other rightmost derivations

LR(k)-Grammars

for example:

(1) $S \to A \mid B \qquad A \to a\,A\,b \mid 0 \qquad B \to a\,B\,b\,b \mid 1$

... is not $LL(k)$ for any k — but $LR(0)$:

Let $\;S \to_R^* \alpha\,X\,w \to \alpha\,\beta\,w\,.\;$ Then $\;\alpha\,\underline{\beta}\;$ is of one of these forms:

$$\underline{A}\,,\; \underline{B}\,,\; a^n\,\underline{a\,A\,b}\,,\; a^n\,\underline{a\,B\,b\,b}\,,\; a^n\,\underline{0}\,,\; a^n\,\underline{1} \qquad (n \geq 0)$$

(2) $S \to a\,A\,c \qquad A \to A\,b\,b \mid b$

... is also not $LL(k)$ for any k — but again $LR(0)$:

Let $\;S \to_R^* \alpha\,X\,w \to \alpha\,\beta\,w\,.\;$ Then $\;\alpha\,\underline{\beta}\;$ is of one of these forms:

$$a\,\underline{b}\,,\; a\,\underline{A\,b\,b}\,,\; \underline{a\,A\,c}$$

LR(k)-Grammars

for example:

(3) $S \to a\,A\,c \qquad A \to b\,b\,A \mid b \qquad$ ... is not $LR(0)$, but $LR(1)$:

Let $\;S \to_R^* \alpha\,X\,w \to \alpha\,\beta\,w\;$ with $\;\{y\} = \text{First}_k(w)\;$ then $\;\alpha\,\beta\,\underline{y}\;$ is of one of these forms:

$$a\,b^{2n}\,\underline{b\,c}\,,\; a\,b^{2n}\,\underline{b\,b\,A}\,c\,,\; \underline{a\,A\,c}$$

(4) $S \to a\,A\,c \qquad A \to b\,A\,b \mid b \qquad$ ... is not $LR(k)$ for any $k \geq 0$:

Consider the rightmost derivations:

$$S \to_R^* a\,b^n\,A\,b^n\,c \to a\,b^n\,\underline{b}\,b^n\,c$$

LR(1)-Parsing

Idea: Let's equip items with 1-lookahead

Definition LR(1)-Item

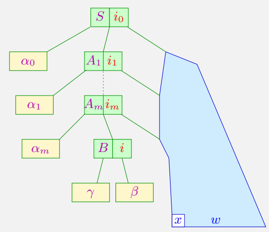
An $LR(1)$-item is a pair $[B \to \alpha \bullet \beta,\; x]$ with

$$x \;\in\; \text{Follow}_1(B) = \bigcup \{\text{First}_1(\nu) \mid S \to^* \mu\,B\,\nu\}$$

Admissible LR(1)-Items

The $LR(1)$-Item $[B \to \gamma \bullet \beta,\; x]$ is *admissable* for $\alpha\,\gamma$ if:

$$S \to_R^* \alpha\,B\,w \qquad \text{with} \qquad \{x\} = \text{First}_1(w)$$



... with $\;\alpha_0 \ldots \alpha_m = \alpha$

The Characteristic LR(1)-Automaton

The set of admissible $LR(1)$-items for viable prefixes is again computed with the help of the finite automaton $\;c(G,1)$.

The automaton $\;c(G,1)$:

States: $LR(1)$-items

Start state: $[S' \to \bullet\, S,\; \$]$

Final states: $\{[B \to \gamma \bullet,\; x] \mid B \to \gamma \;\in\; P, x \;\in\; \text{Follow}_1(B)\}$

Transitions:

(1) $\;([A \to \alpha \bullet X\,\beta,\; x], X, [A \to \alpha\,X \bullet \beta,\; x]), \quad X \;\in\; (N \cup T)$

(2) $\;([A \to \alpha \bullet B\,\beta,\; x], \epsilon,\; [B \to \bullet\,\gamma,\; x']), \quad A \to \alpha\,B\,\beta\,,\; B \to \gamma \;\in\; P,$
$\quad x' \;\in\; \text{First}_1(\beta) \odot_1 \{x\}$

This automaton works like $c(G)$ — but additionally manages a 1-prefix from Follow_1 of the left-hand sides.

The Canonical LR(1)-Automaton

The canonical $LR(1)$-automaton $LR(G,1)$ is created from $c(G,1)$, by performing arbitrarily many ϵ-transitions and then making the resulting automaton deterministic ...

But again, it can be constructed directly from the grammar; analoguously to $LR(0)$, we need the ϵ-closure δ_ϵ^* as a helper function:

$$\delta_\epsilon^*(q) = q \cup \{[C \to \bullet\,\gamma,\; x] \mid \;\; [A \to \alpha \bullet B\,\beta',\; x'] \in q,\;\; B \to^* C\,\beta\,,\;\; C \to \gamma \;\in\; P\,,$$
$$x \in \text{First}_1(\beta\,\beta') \odot_1 \{x'\}\}$$

Then, we define:

States: Sets of $LR(1)$-items;

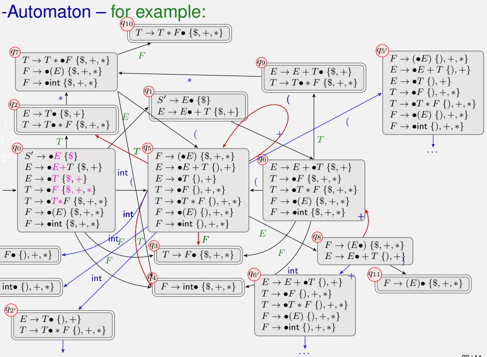
Start state: $\delta_\epsilon^*\;\{[S' \to \bullet\, S,\; \$]\}$

Final states: $\{q \mid [A \to \alpha \bullet,\; x] \;\in\; q\}$

Transitions: $\delta(q, X) = \delta_\epsilon^*\;\{[A \to \alpha\,X \bullet \beta,\; x] \mid [A \to \alpha \bullet X\,\beta,\; x] \in q\}$

The Canonical LR(1)-Automaton – for example:

$$S' \;\to\; E$$
$$E \;\to\; E + T \;\mid\; T$$
$$T \;\to\; T * F \;\mid\; F$$
$$F \;\to\; (\,E\,) \;\mid\; \text{int}$$



The Canonical LR(1)-Automaton

Discussion:

- In the example, the number of states was almost doubled
  ... and it can become even worse
- The conflicts in states q_1, q_2, q_9 are now resolved !
  e.g. we have:

$$q_9: \quad \begin{array}{l} E \to E + T \bullet \;\{\$, +\} \\ T \to T \bullet * F \;\{\$, +, *\} \end{array}$$

with:

$$\{\$, +\} \cap (\text{First}_1(* \, F) \odot_1 \{\$, +, *\}) \;=\; \{\$, +\} \cap \{*\} = \emptyset$$

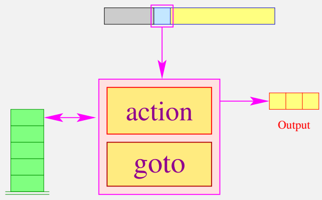
The Action Table:

During practical parsing, we want to represent states just via an integer id. However, when the canonical $LR(1)$-automaton reaches a final state, we want to know *how to reduce/shift*. Thus we introduce...

The construction of the action table:

Type: action : $Q \times T \to LR(0)$-Items $\cup$ {s, error}
Reduce: action$[q, w] = [A \to \beta \bullet]$ if $[A \to \beta \bullet, w] \in q$
Shift: action$[q, w] = s$ if $[A \to \beta \bullet b\,\gamma, a] \in q,\ w \in$ First$_1(b\,\gamma) \odot_1 \{a\}$
Error: action$[q, w] =$ error else

The LR(1)-Parser:



- The goto-table encodes the transitions:

$$\text{goto}[q, X] = \delta(q, X) \in Q$$

- The action-table describes for every state q and possible lookahead w the necessary action.

The LR(1)-Parser:

The construction of the $LR(1)$-parser:

States: $Q \cup \{f\}$ (f fresh)
Start state: q_0
Final state: f
Transitions:

Shift: $(p, a, p\,q)$ if $a = w$, $s = \text{action}[p, a]$, $q = \text{goto}[p, a]$

Reduce: $(p\,q_1 \ldots q_{|\beta|}, \epsilon, p\,q)$ if $q_{|\beta|} \in F$, $[A \to \beta \bullet] = \text{action}[q_{|\beta|}, w]$, $q = \text{goto}[p, A]$

Finish: $(q_0\,p, \epsilon, f)$ if $[S' \to S \bullet, \$] \in p$

with $LR(G, 1) = (Q, T, \delta, q_0, F)$ and the lookahead w.

The LR(1)-Parser:

Possible actions are:

shift // Shift-operation
reduce $(A \to \gamma)$ // Reduction with callback/output
error // Error

... for example:

$$\begin{array}{lcl} S' & \to & E \\ E & \to & E + T^{\,0} \mid T^{\,1} \\ T & \to & T * F^{\,0} \mid F^{\,1} \\ F & \to & (\,E\,)^{\,0} \mid \texttt{int}^{\,1} \end{array}$$

action	\$	int	(	)	+	*
q_1	$S', 0$				s	
q_2	$E, 1$				$E, 1$	s
q'_2				$E, 1$	$E, 1$	s
q_3	$T, 1$				$T, 1$	$T, 1$
q'_3				$T, 1$	$T, 1$	$T, 1$
q_4	$F, 1$				$F, 1$	$F, 1$
q'_4				$F, 1$	$F, 1$	$F, 1$
q_9	$E, 0$				$E, 0$	s
q'_9				$E, 0$	$E, 0$	s
q_{10}	$T, 0$				$T, 0$	$T, 0$
q'_{10}				$T, 0$	$T, 0$	$T, 0$
q_{11}	$F, 0$				$F, 0$	$F, 0$
q'_{11}				$F, 0$	$F, 0$	$F, 0$

The Canonical LR(1)-Automaton

In general: We identify two conflicts for a state $q \in Q$:

Reduce-Reduce-Conflict:

q: $\boxed{\begin{array}{l} A \to \gamma \bullet,\ x \\ A' \to \gamma' \bullet,\ x \end{array}}$ with $A \neq A' \vee \gamma \neq \gamma'$

Shift-Reduce-Conflict:

q: $\boxed{\begin{array}{l} A \to \gamma \bullet,\ x \\ A' \to \alpha' \bullet a\,\beta,\ y \end{array}}$ with $a \in T$ und $x \in \{a\} \odot_k$ First$_k(\beta) \odot_k \{y\}$.

Such states are now called $LR(1k)$-unsuited

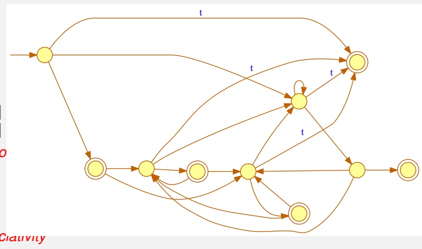
Theorem:
A reduced contextfree grammar G is called $LR(k)$ iff the canonical $LR(k)$-automaton $LR(G, k)$ has no $LR(k)$-unsuited states.

Precedences

Many parser generators give the chance to fix Shift-/Reduce-Conflicts by patching the action table either by hand or with *token precedences*.

... for example:

$$\begin{array}{lcl} S' & \to & E^{\,0} \\ E & \to & E + E^{\,0} \\ & \mid & E * E^{\,1} \\ & \mid & (\,E\,)^{\,2} \\ & \mid & \texttt{int}^{\,3} \end{array}$$



Shift-/Reduce Conflict in state 8:

$$\begin{array}{l} [E \to E \bullet + E^{\,0} \quad] \\ [E \to E + E \bullet^{\,0} \quad , +] \end{array}$$

$< \gamma\, E + E\, , + \omega >$ $\Rightarrow$ *Asso*

Shift-/Reduce Conflict in state 7:

$$\begin{array}{l} [E \to E \bullet * E^{\,1} \quad] \\ [E \to E * E \bullet^{\,1} \quad , *] \end{array}$$

$< \gamma\, E * E\, , * \omega >$ $\Rightarrow$ *Associativity*

Shift-/Reduce Conflict in states 8, 7:

$*$ *right associative*
$+$ *left associative*

$[E \to E \bullet * E^{\,1} \quad]$

What if precedences are not enough?

Example (very simplified lambda expressions):

$$\begin{array}{lcl} E & \to & (\,E\,)^{\,0} \mid \text{ident}^{\,1} \mid L^{\,2} \\ L & \to & \langle\text{args}\rangle \Rightarrow E^{\,0} \\ \langle\text{args}\rangle & \to & (\,\langle\text{idlist}\rangle\,)^{\,0} \mid \text{ident}^{\,1} \\ \langle\text{idlist}\rangle & \to & \langle\text{idlist}\rangle\ \text{ident}^{\,0} \mid \text{ident}^{\,1} \end{array}$$

E rightmost-derives these forms among others:

$(\ \underline{\text{ident}}\ ),\ (\ \underline{\text{ident}}\ ) \Rightarrow \text{ident}\ , \ldots \quad \Rightarrow \quad$ at least $LR(2)$

Naive Idea:
poor man's $LR(2)$ by combining the tokens) and $\Rightarrow$ during lexical analysis into a single token $)\!\Rightarrow$.

⚠ in this case obvious solution, but in general not so simple

What if precedences are not enough?

In practice, $LR(k)$-parser generators working with the lookahead sets of sizes larger then $k = 1$ are not common, since computing lookahead sets with $k > 1$ blows up exponentially. However,

- there exist several practical $LR(k)$ grammars of $k > 1$, e.g. Java 1.6+ ($LR(2)$), ANSI C, etc.
- often, more lookahead is only exhausted locally
- should we really give up, whenever we are confronted with a Shift-/Reduce-Conflict?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Theorem: $LR(k)$-to-$LR(1)$
Any $LR(k)$ grammar can be directly transformed into an equivalent $LR(1)$ grammar.

LR(2) to LR(1)

... Example:

$$\begin{array}{lcl} S & \to & A\,b\,b^{\,0} \mid B\,b\,c^{\,1} \\ A & \to & a\,A^{\,0} \mid a^{\,1} \\ B & \to & a\,B^{\,0} \mid a^{\,1} \end{array}$$

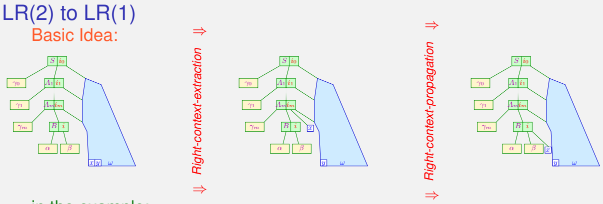
S rightmost-derives one of these forms:

$a^n\underline{a}\,b\,b\ ,\ a^n\underline{a}\,b\,c\ ,\ a^n\underline{a\,A}\,b\,b\ ,\ a^n\underline{a\,B}\,b\,c,\ \underline{A}\,b\,b\ ,\ \underline{B}\,b\,c \quad \Rightarrow \quad LR(2)$

in $LR(1)$, you will have Reduce-/Reduce-Conflicts between the productions $A, 1$ and $B, 1$ under lookahead b

LR(2) to LR(1)

Basic Idea:



in the example:

Right-context is already extracted, so we only perform *Right-context-propagation*:

$$\begin{array}{lcl} S & \to & A\,b\,b^{\,0} \mid B\,b\,c^{\,1} \\ A & \to & a\,A^{\,0} \mid a^{\,1} \\ B & \to & a\,B^{\,0} \mid a^{\,1} \end{array} \quad \Rightarrow \quad \begin{array}{lcl} S & \to & \langle A\,b\rangle\,b^{\,0} \mid \langle B\,b\rangle\,c^{\,1} \\ \langle A\,b\rangle & \to & a\,\langle A\,b\rangle^{\,0} \mid a\,b^{\,1} \\ \langle B\,b\rangle & \to & a\,\langle B\,b\rangle^{\,0} \mid a\,b^{\,1} \\ A & \to & a\,A^{\,0} \mid a^{\,1} \\ B & \to & a\,B^{\,0} \mid a^{\,1} \end{array}$$

unreachable

LR(2) to LR(1)

Example cont'd:

$$\begin{array}{lcl} S & \rightarrow & A'\,b^{0} \mid B'\,c^{1} \\ A' & \rightarrow & a\,A'^{0} \mid a\,b^{1} \\ B' & \rightarrow & a\,B'^{0} \mid a\,b^{1} \end{array}$$

S rightmost-derives one of these forms:

$$a^n\,\underline{a\,b}b\;,\;a^n\,\underline{a\,b}c\;,\;a^n\,\underline{a\,A'}b\;,\;a^n\,\underline{a\,B'}c,\;\underline{A'}b\;,\;\underline{B'}c \quad\Rightarrow\quad LR(1)$$

LR(2) to LR(1)

Example 2:

$$\begin{array}{lcl} S & \rightarrow & b\,S\,S^{0} \\ & \mid & a^{1} \\ & \mid & a\,a\,c^{2} \end{array}$$

S rightmost-derives these forms among others:

$$\underline{b\,S\,S},\; b\,S\,\underline{a},\; b\,S\,\underline{a\,a\,c},\; b\,\underline{a}\,a,\; b\,\underline{a\,a\,c}\,a,\; b\,\underline{a}\,a\,a\,c,\; b\,\underline{a\,a\,c}\,a\,a\,c,\;\ldots \;\Rightarrow\; \text{min. } LR(2)$$

in $LR(1)$, you will have (at least) Shift-/Reduce-Conflicts between the items $[S\rightarrow a\bullet, a]$ and $[S\rightarrow a\bullet ac]$

$[S\rightarrow a]$'s right context is a nonterminal ⇒ perform *Right-context-extraction*

$$\begin{array}{lcl} S & \rightarrow & b\,S\,S^{0} \\ & \mid & a^{1} \\ & \mid & a\,a\,c^{2} \end{array} \quad\Rightarrow\quad \begin{array}{lcl} S & \rightarrow & b\,S\,a\,\langle a/S\rangle^{0} \mid b\,S\,b\,\langle b/S\rangle^{0'} \\ & \mid & a^{1} \mid a\,a\,c^{2} \\ \langle a/S\rangle & \rightarrow & \epsilon^{0} \mid a\,c^{1} \\ \langle b/S\rangle & \rightarrow & S\,S^{0}S\,a\,\langle a/S\rangle^{0} \mid S\,b\,\langle b/S\rangle^{0'} \end{array}$$

LR(2) to LR(1)

Example 2 cont'd:

$[S\rightarrow a]$'s right context is now terminal a ⇒ perform *Right-context-propagation*

$$\begin{array}{lcl} S & \rightarrow & b\,S\,a\,\langle a/S\rangle^{0} \\ & \mid & b\,S\,b\,\langle b/S\rangle^{0'} \\ & \mid & a^{1} \mid a\,a\,c^{2} \\ \langle a/S\rangle & \rightarrow & \epsilon^{0} \mid a\,c^{1} \\ \langle b/S\rangle & \rightarrow & S\,a\,\langle a/S\rangle^{0} \mid S\,b\,\langle b/S\rangle^{0'} \end{array} \quad\Rightarrow\quad \begin{array}{lcl} S & \rightarrow & b\,\langle Sa\rangle\,\langle a/S\rangle^{0} \\ & \mid & b\,S\,b\,\langle b/S\rangle^{0'} \\ & \mid & a^{1} \mid a\,a\,c^{2} \\ \langle a/S\rangle & \rightarrow & \epsilon^{0} \mid a\,c^{1} \\ \langle b/S\rangle & \rightarrow & \langle Sa\rangle\,\langle a/S\rangle^{0} \mid S\,b\,\langle b/S\rangle^{0'} \\ \langle Sa\rangle & \rightarrow & b\,\langle Sa\rangle\,\langle a/S\rangle\,a\langle\langle a/S\rangle\,a\rangle^{0} \\ & \mid & b\,S\,b\,\langle b/S\rangle\,a\langle\langle b/S\rangle\,a\rangle^{0'} \\ & \mid & a\,a^{1} \mid a\,a\,c\,a^{2} \\ \langle\langle a/S\rangle a\rangle & \rightarrow & a^{0} \mid a\,c\,a^{1} \\ \langle\langle b/S\rangle a\rangle & \rightarrow & \langle Sa\rangle\langle a/S\rangle\,a\langle\langle a/S\rangle a\rangle^{0} \mid S\,b\,\langle b/S\rangle\,a\langle\langle b/S\rangle a\rangle \end{array}$$

LR(2) to LR(1)

Example 2 finished:

With fresh nonterminals we get the final grammar

$$\begin{array}{lcl} S & \rightarrow & b\,S\,S^{0} \\ & \mid & a^{1} \\ & \mid & a\,a\,c^{2} \end{array} \quad\Rightarrow\quad \begin{array}{lcl} S & \rightarrow & b\,C\,A,^{0} \mid b\,S\,b\,B,^{1} \mid a^{2} \mid a\,a\,c^{3} \\ A & \rightarrow & \epsilon^{0} \mid a\,c^{1} \\ B & \rightarrow & C\,A^{0} \mid S\,b\,B^{1} \\ C & \rightarrow & b\,C\,D^{0} \mid b\,S\,b\,E^{1} \mid a\,a^{2} \mid a\,a\,c\,a^{3} \\ D & \rightarrow & a^{0} \mid a\,c\,a^{1} \\ E & \rightarrow & C\,D^{0} \mid S\,b\,E^{1} \end{array}$$