Learning in Games Seminar

Introduction to Topics – 29. March 2023

Chair of Decision Sciences & Systems

Agenda

- Seminar format
- Background information
  - Recap: Games and Nash equilibria
  - A brief taxonomy of games
  - What is “learning in games” anyway?
  - A short primer on (multi-agent) reinforcement learning
- Topic assignments
- Q&A

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Format

Each student will be assigned an individual topic and prepare a presentation as well as a short summary paper

Presentation

45 min presentation
of topic

Handout

should be
shared at
least 3 days
before
presentation

Short handout (extended abstract) of
maximal 2 pages

Quiz

reading
handout
should
suffice for
full score

Mini-quiz for the audience: < 1 min
with MC or single-word responses

- We will have **biweekly meetings** throughout the semester with two topics being presented in each meeting
- Attendance of all meetings is mandatory
  - interaction with the other students' work is expected: E.g., answering prepared mini-quizzes

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What is a game?

- $G = (N, A, u)$
- Set of players $\mathcal{I} = \{1, \dots, N\}$
- Each player has a set of actions: A_i
- Utility function: $u_i : A_1 \times \cdots \times A_N \to \mathbb{R}$

Battle of the Sexes Game

B \ A	B Fight	B Ballet
A Fight	2 / 3	0 / 0
A Ballet	0 / 0	3 / 2

Outcome $u_i(a_1, \dots, a_N)$ for player i depends on **all** players‘ actions.

Strategic behavior

- How should a player i choose his or her action a_i?
- Goal: maximize expected utility u_i.
  - A player that achieves this given all the information that is available to them is called *rational.*
  - Important: Information about other players, and information about other players‘ information about other players, ... („common knowledge“)
- Optimal play often involves randomization. A *mixed strategy* π is a probability distribution (i.e., a vector) over all available actions.
  - For the resulting space of mixed-strategies, we write Π or ΔA.

Notational conventions

- a_i, π_i, u_i, ... describe the action/strategy/utility/... of a single player i.
- a, π, u describe vectors over all players' actions/strategies/utilities. Such vectors are called action-/strategy-/... *profiles*.
- a_{-i}, π_{-i}, u_{-i} describe the partial profiles for all players, except player i.
- Utility of a strategy profile, means the expected utility of resulting action profiles:

$$u_i(\pi_i, \pi_{-i}) \coloneqq \mathbb{E}_{a \sim \pi}[u_i(a_i, a_{-i})]$$

Nash equilibrium

- A strategy profile π^* is a Nash equilibrium (NE), if and only if no single player can improve his or her expected utility by unilaterally changing the strategy:

$$\forall i \text{ and } \forall \pi_i \in \Pi_i: \quad u_i(\pi_i, \pi^*_{-i}) \leq u_i(\pi^*_i, \pi^*_{-i})$$

- At least one Nash equilibrium exists in every finite game.

- Nevertheless, NE are generally hard to compute.

Are Nash equilibria always the goal?

Prisoner's dilemma payoff matrix

B / A	B silent	B betrays
A silent	-1 (B) / -1 (A)	0 (B) / -3 (A)
A betrays	-3 (B) / 0 (A)	-2 (B) / -2 (A)

- NE are often hard to compute, but there are other equilibrium notions, like (coarse) correlated equilibria, that are easier to attain.
- Besides being hard to compute, NE may not always lead to desirable outcomes.
  → "Social Dilemma"
- Sometimes, these problems can be circumvented in repeated games.

Types of games – qualitative dimensions

Number of players

- Special case: 2
- “few”
- Many/infinite/continuum

Stateless vs. stateful

- Normal form
- Extensive form
- Repeated game
- Simultaneous moves vs. sequential moves

Types of utility functions

- Zero-Sum
- Nonzero-sum

Number of actions

- Finite
- Countably infinite
- Continuum

Observability of information

- **Complete vs. incomplete**
  - *Complete information:* Players know the rules of the game and others’ payoff functions (but may possibly not know about past moves of other players or outcomes of chance events).
  - *Complete information:* The structure of the game and the players’ utility functions are commonly known (but players may not see all the moves made by other players).
- **Perfect vs. imperfect information**
  - *Perfect information:* Players can observe all events and the full ‘state’ of the game (but may not know about opponents’ goals).
  - *Imperfect information:* Games where some aspect of play is hidden from players (e.g., poker).

Special Case 1

Matrix games

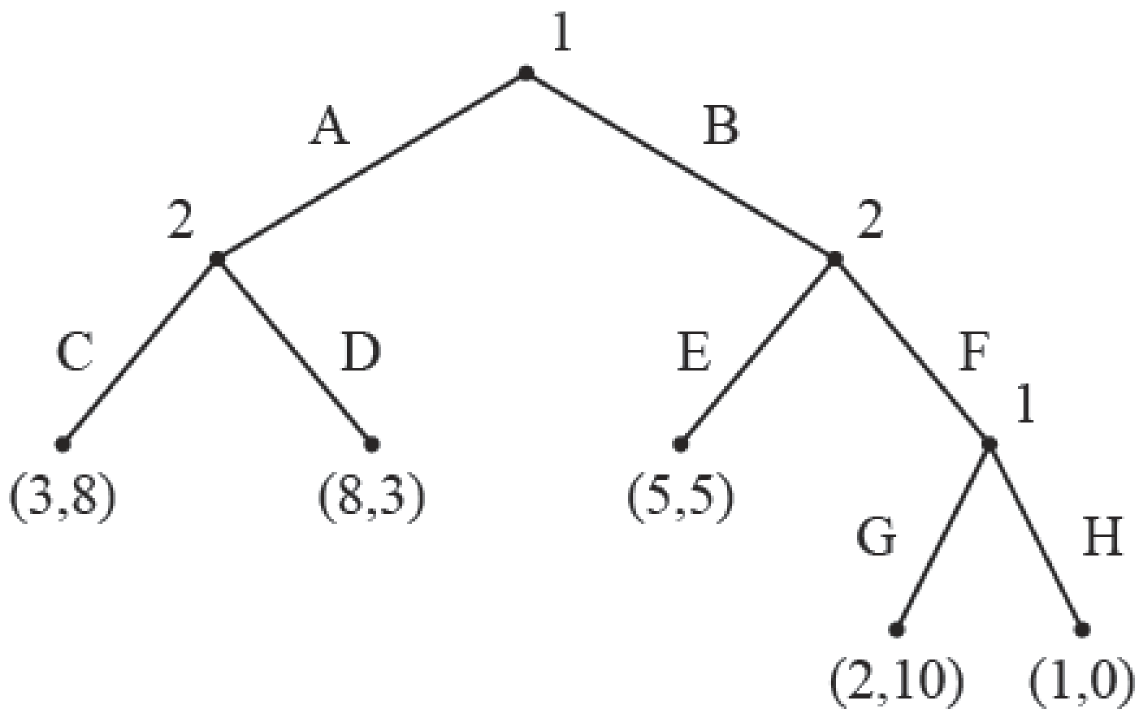
- Finitely many players
- Finitely many actions
- A single, simultaneous move
- Fixed outcomes, no randomness in utility functions
- (Both, complete and perfect information)

B A	B Fight	B Ballet
A Fight	2 3	0 0
A Ballet	0 0	3 2

Special Case 2

Extensive form games

- Sequential actions by players, leading to a "game tree"
- Can have perfect or imperfect information



Special Case 3

Two-player zero-sum games

- Everything that's good for P1 is bad for P2, can be written as

$$u_1 = u \qquad\qquad u_2 = -u$$

- The *minimax* theorem applies:

$$\max_{\pi_1} \min_{\pi_2} u(\pi_1, \pi_2) = \min_{\pi_2} \max_{\pi_1} u(\pi_1, \pi_2)$$

- This makes it easier (in theory) to find and understand NE
- Examples: Two-player board games (Chess, Go, …)
- However: Game can still be extremely large and hard to solve in practice!

Image: James Wainscoat, https://unsplash.com photos/b7MZ6iGIoSI

Special Case 4

Mean field games

- Games with very large populations (e.g., 100s, 1000s, millions of players).
- Each individual agent will have a negligible impact on others.
- Idea: model others as a “continuum” rather than individual actors.
- Name inspired by mean-field theory in physics.
- Examples:
  - Choosing a route on your commute to avoid a traffic jam.
  - Behavior in saturated markets.

Learning in games

> Most of non-cooperative game theory has focused on equilibrium in games […]. This raises the question of when and why we might expect that observed play in a game will correspond to one of these equilibria. One traditional explanation of equilibrium is that it results from analysis and introspection by the players in a situation where the rules of the game, the rationality of the players, and the players' payoff functions are all common knowledge. […]
>
> This book develops the alternative explanation that equilibrium arises as the longrun outcome of a process in which less than fully rational players grope for optimality over time.

- Fudenberg & Levine, The Theory of Learning in Games (1999), page 1

Learning in games

- Most common setting: Learning agents play the game against each other iteratively, in *self-play.*
- Agents *update* their strategies over time, dependent on observed outcomes, in order to improve their own expected utility.
- Update strategies:
  - "Best-response dynamics",
  - improvement updates, often based on (regularized) gradient dynamics, or regret minimization,
  - solving a "meta game", of strategies encountered before (EGTA/PSRO),
  - reinforcement-learning-based updates.

Supervised machine learning

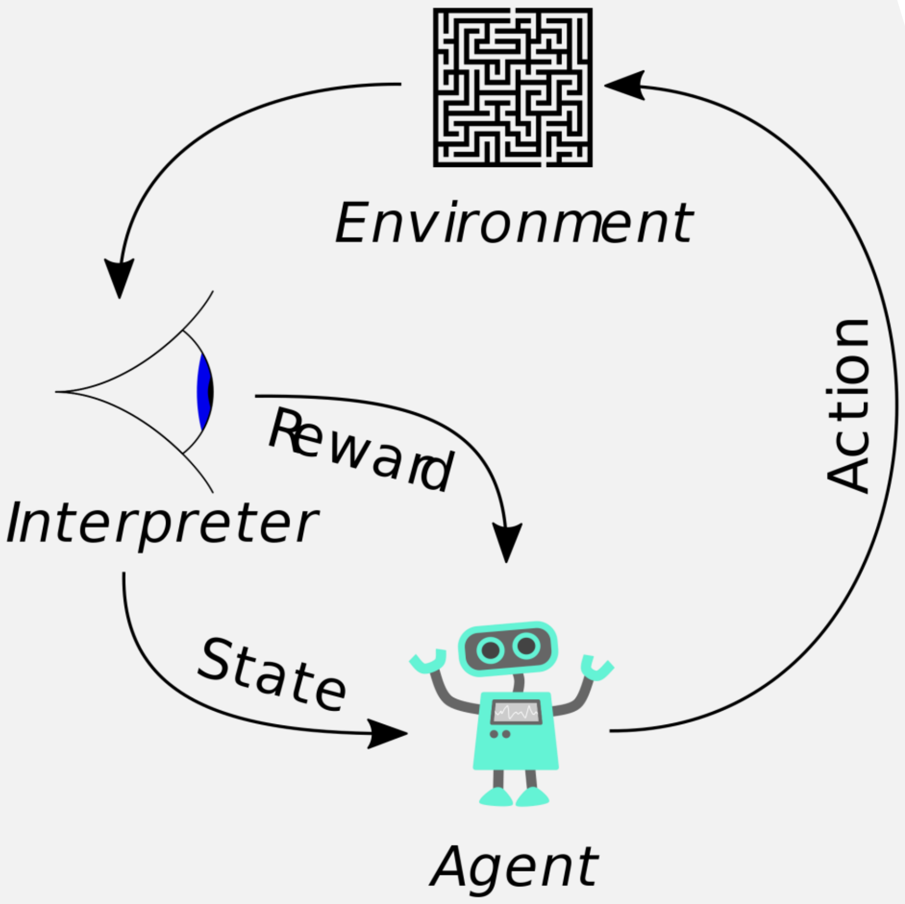
Input: Data set of observations (x_i, y_i)

x_i: features y_i: labels

Goal: Find a *model* $\hat{f}$ such that

$$\hat{f}(x_i) \approx y_i$$

(Single-agent) reinforcement learning



At time *t:*

- agent observes **state** s_t
- Agent chooses **action** $a_t = \pi(s_t)$

Based on a_t, agent receives a **reward** r_{t+1}

Goal: Choose policy π to maximize expected future **return**

$$R_t = \sum_{\tau=1}^{T} \gamma^\tau \, r_{t+\tau}$$

(with $\gamma \in (0,1)$: „discount rate“)

Note: future state depends on current actions!

In practice: randomized actions: $\pi(a_t, s_t)$ = probability of playing a_t when observing s_t

Image: Wikipedia

Formal model for (SA)RL: Markov decision process (MDP)

At each time step t:

- Agent chooses a_t from a set of actions A_t
- **Stationary** state transition distribution given s_t and a_t: $P(s_{t+1} | s_t, a_t)$
- Rewards r_t associated with state transitions

→ Markov property:
„The future is independent of the past, given the present“

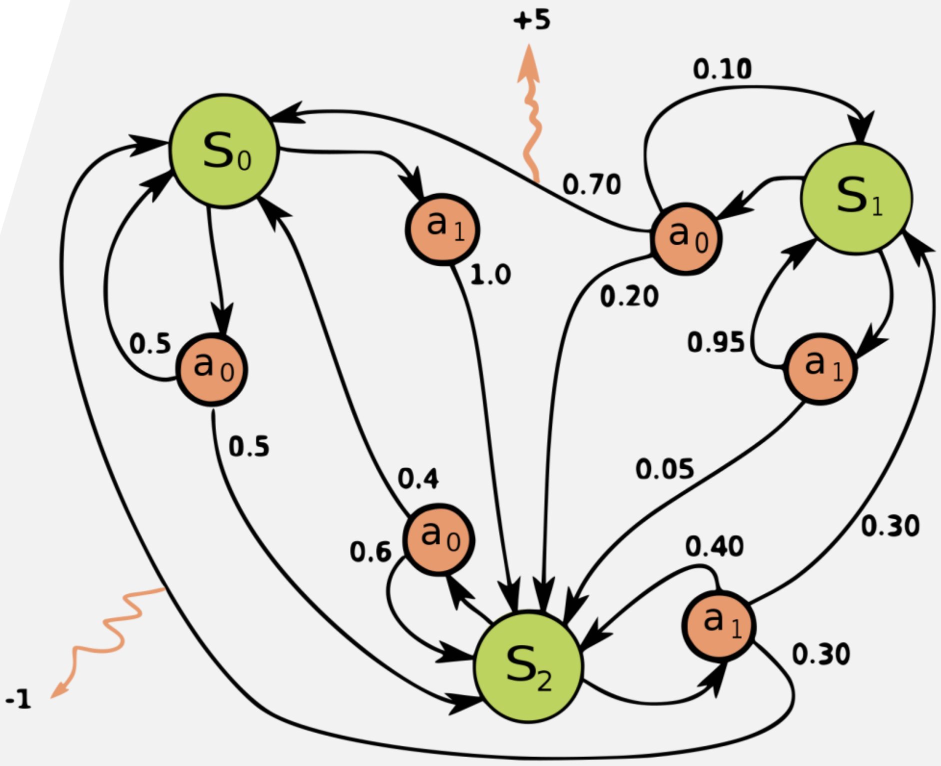


Image: Wikipedia

How to “train” π in (SA)RL?

Data: sampled transitions: $(s_t, a_t, r_{t+1}, s_{t+1})$

Value Iteration

- Estimate the *value* of a state $v(s)$, choose the action that maximizes the expected value of s_{t+1}.
- or directly estimate *state-action values* $q(s, a)$ („Q-Learning“).

Policy Iteration

- Rather than learning values, act directly on the policy function $\pi(s, a)$.
- Most common: „policy gradient“: Estimate $\nabla_\pi \mathbb{E}[R_t]$ from data, then perform gradient ascent.

For MDPs (and some additional details), both converge to optimal policy, in theory.

Practical problem: computational tractability!

(SA)RL: Approximation methods

- Tabular methods:
  - Keep track of q-values for all combinations (s, a), model policies as explicit probability vectors $\pi(s, a) = P(a|s)$.
    → Only feasible for small state and action spaces.
- Approximation Methods:
  - Learn statistical models to represent one or more of v, q, π.
  - Usually using neural networks → „deep reinforcement learning“.
- Examples of deep RL:
  - Deep Q-learning (DQN): Model $q(s, a)$ via a neural net (+ extra tricks).
  - DDPG (deep deterministic policy gradient): Model $q(s, a)$ as one neural net, then use policy iteration on second neural net $\pi(s, a)$ based on *predicted* improvement $\nabla_\pi \, q(s, a)$ → „actor-critic method“.

Multi-agent RL (MARL)

- *Markov game* as a generalization of MDP
- Main Challenge: environment loses its stationarity: state transitions now depend on other agents‘ actions!
  → convergence results from SA-RL break
- Cooperative MARL:
  - Agents share a common utility function, need to learn to work together
- Competitive MARL:
  - Agents have individual utility functions, only interested in their own rewards
- Mixed settings (e.g., team games) are also possible

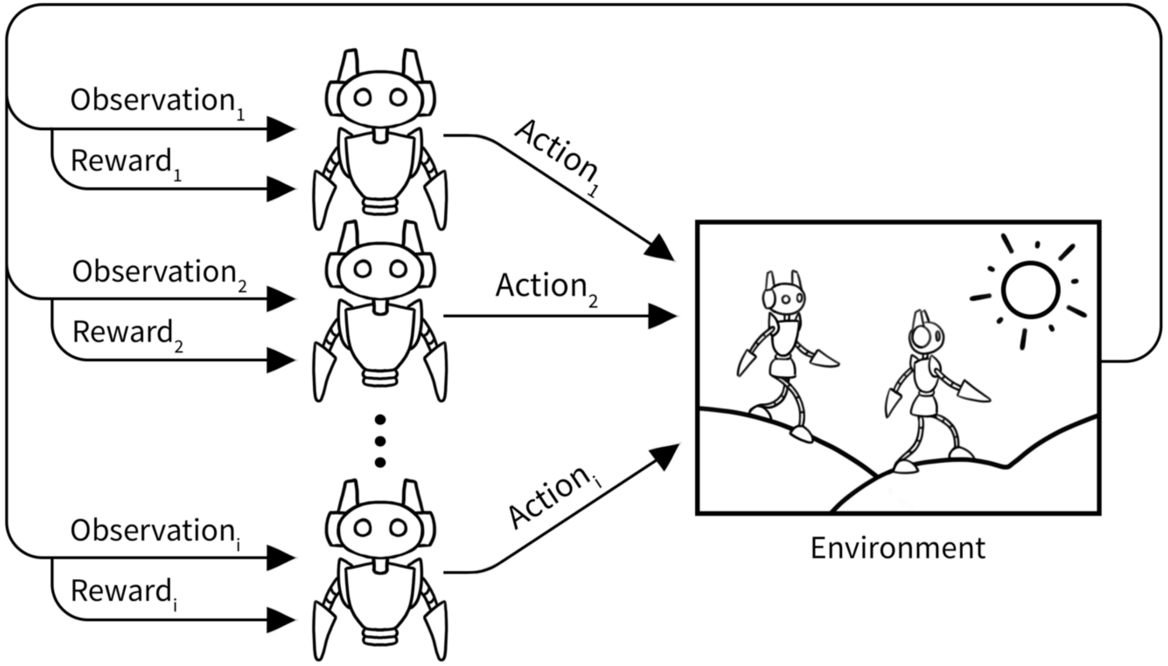


Image: Justin Terry

What's the goal in competitive MARL?

- Equilibrium?
  - For two-player zero-sum games: Yes!
  - Otherwise…?
- Super-human performance?
- Social welfare?

Further Reading: Shoham et al. (2007): If multi-agent learning is the answer, what is the question?

General literature recommendations

- Fudenberg, et al. (1998): *The theory of learning in games*
  - Foundation for equilibrium theory, replicator dynamics and fictious play, normal and extensive form games
  - Can be borrowed at chair, TUM library has a few hard-copies
- Young (2004): *Strategic learning and its limits*
  - Regret and no-regret learning, equilibrium concepts, fictious play
  - Can be borrowed at chair
- Shalev-Shwartz (2011): *Online learning and online convex optimization*
  - Survey paper on online learning that introduces , among other, FTL and FTRL, …
  - Available online
- Nisan, Roughgarden, Tardos, Vazirani (2007): *Algorithmic Game Theory*
  - Collection of guest chapters on different topics in AGT, e.g., complexity, learning, applications, …
- Sutton & Barto (2018): *Reinforcement Learning. An Introduction (2nd edition)*
  - The standard reference for (single-agent) reinforcement learning.
  - Latest edition available for free at http://www.incompleteideas.net/book/the-book.html

Practical Hint:

Accessing paywalled papers

- eAccess is a service provided by the TUM university library. Using eAccess, you can access many scientific papers online that would usually be behind a paywall.
  See https://login.eaccess.tum.edu/login

- If you still cannot access a paper/book you need:
  1. Check the TUM university library catalog.
     Some books are only available as hard copies
  2. Ask your advisor for help.

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Course Content

Available Topics

01	Normal Form Games & Equilibria
02	Regret Matching & Convergence
03	Zero-Sum, Fictitious Play, MiniMax
04	MDP + Q-Learning
05	Policy Gradients
06	Stochastic Games + Solution Concepts + Shapley Algorithm
07	Lemke-Howson Algorithm
08	Multiplicative Weights & Replicator Dynamics
09	Regret Policy Gradients
10	Collusion
11	Policy Space Response Oracles
12	Sophisticated/Bayesian Learning
13	Inverse Reinforcement Learning
14	Neural Equilbirum Solvers

Topic 01

Normal Form Games & Equilibria

- Introduce basic concepts from Game Theory
  - Normal Form Game
  - Strategy, Dominance, Optimality
  - Solution Concepts:
    - Nash Equilibrium
    - Correlated Equilibrium
    - Coarse Correlated Equilibrium
    - ... others ?
- Illustrate concepts with examples
  - Prisoners’ Dilemma, Matching Pennies, Chicken, etc.

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Topic 02

Regret Matching & Convergence

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- The regret matching algorithm (Hart & Mas-Colell, 2000) seeks to minimize regret about its actions.
  - Regret of not having chosen an action: Difference between the utility of that action and the utility of the action we actually chose, with respect to the fixed choices of other players.
  - It learns from past behavior by favoring actions that have resulted in positive outcomes and avoiding actions that have resulted in negative ones.
- Is the foundation for modern poker bots
- Considers simple normal-form games
- The averaged strategy then converges to a (correlated) equilibrium

Zero-Sum, Fictitious Play, MiniMax

- MiniMax Value Iteration:
  - Determine the best worst-case scenario
  - Solution corresponds to a Nash Equilibrium in 2-Player Zero-Sum games
- Fictitious Play:
  - Agents assume that opponents play stationary strategies
  - Play the best-response against the frequency of opponents play
  - Convergence to the Nash Equilibrium in 2-Player Zero-Sum games
  - Variety of adaptions of this learning procedure

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Literature: Fudenberg and Levine (1998)

Topic 04

MDP + Q-Learning

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- A Markov decision process is the mathematical framework of many fields, such as
  - Reinforcement learning
  - Planning
  - Control theory
  - … and many more
- Introduce MDP mathematics and concepts
  - State, action, transition, reward, policy, etc.
  - What does it mean to “solve” an MDP?
- Discuss Q-learning as one example of an approximate MDP solver
  - $Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \cdot \left[R(s_t, a_t) + \gamma \cdot \max_a Q(s_{t+1}, a) - Q(s_t, a_t)\right]$

Topic 05

Policy Gradients

- Prerequisite: MDPs
- Policy gradient theorem (PGT):

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log\big(\pi_\theta(s,a)\big) \cdot Q^{\pi_\theta}(\mathrm{s},\mathrm{a})\right]$$

- Many solution methods build upon PGT
- Your task:
  - Understand, explain, illustrate
  - Discuss advantages and disadvantages of policy gradient methods

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Stochastic Games + Solution Concepts + Shapley Algorithm

- Stochastic games (Shapley, 1953) are a natural extension of MDPs to include multiple agents. We want to look at the basic definitions and examples:
  - Stochastic games contain both MDPs and matrix games as subsets of the framework
  - Similar to normal-form games, there are different types (e.g., zero-sum stochastic games)
  - Only for some settings equilibria solutions are known to exist
- The Shapley algorithm can be used to compute equilibria in zero-sum stochastic games
  - The algorithm is based on the proof of the equilibrium existence (Shapley, 1953)
  - The method is similar to value iteration for MDPs (Bowling & Veloso, 2000)

Topic 07

Lemke-Howson Algorithm

- The Lemke-Howson algorithm (Lemke & Howson, 1964) is an algorithm that computes a Nash equilibrium of two-player matrix games.
- It doesn't scale well but does solve the game exactly.
  - Worst case: Number of operations may be exponential in the number of pure strategies.
- It resembles the simplex algorithm (from linear programming).

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Multiplicative Weights & Replicator Dynamics

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- Replicator Dynamics
  - Motivated by the evolution of biological processes
  - Does not necessarily converge to a stable state (i.e., a Nash equilibrium)
- Multiplicative Weights Update:
  - Discrete-time variant of Replicator Dynamics
  - Assign each action a weight
  - Update the weight multiplicatively in each iteration
  - Converges to the Nash Equilibrium in time-average strategies of 2-Player Zero-Sum games
  - Actual strategies diverge from the equilibrium

- Literature: Fudenberg and Levine (1998), Bailey and Piliouras (2018)

Topic 09

Regret Policy Gradients

- Learning parameterized policies (e.g., neural networks) in RL.
- Introduced by Srinivasan et al. (2018) from Deepmind.
- Combination of
  - standard gradient-based learning (following the policy gradient uphill) and
  - regret minimization.
- They consider partially-observable multiagent environments and the algorithm is implemented in Openspiel.

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Topic 10

Collusion

- In general, learning algorithms do not need to converge to Nash equilibria
- Sometimes the process can even lead to undesirable outcomes such as collusion, e.g., firms charge supracompetitive prices (Calvano et al., 2020)
- On the other hand, the choice of algorithms can change the game.
  And collusive outcomes might actually just be equilibrium strategies in the new game (den Boer et al, 2022)
- Therefore, choosing the right learning algorithm and model is crucial.

Topic 11

Policy Space Response Oracles

- Considers the meta-game, where agents are trained via RL in an inner loop and we apply game-theoretic reasoning in the outer loop
- Goal is to arrive at more robust strategies by training a collection of low-level controllers
- Subsumes classical approaches such as independent learning, iterated best response, double oracle, and fictitious play.
- Applying this is costly, but opens up the path to reason about more practical approaches
- „A Unified Game-Theoretic Approach to Multiagent Reinforcement Learning", Lanctot et al., 2017 in Advances in neural information processing systems, 30

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Sophisticated/Bayesian Learning

- Sophisticated learning
  - Many learning algorithms fail to identify patterns (like cycles)
  - Sophisticated Learning explicitly attempts to find those patterns
- Bayesian learning
  - Agents have prior knowledge about the game (opponents' strategies, random state)
  - Represent different beliefs as different models
  - If priors are consistent with the true model, the strategies converge to the true model, which implies convergence to an equilibrium

Literature: Fudenberg and Levine (1998), Wu et al. (2022)

Topic 13

Inverse Reinforcement Learning

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- Idea
  - Assume that optimal actions are already observed by agents
  - Find the reward function that leads to these actions
- Three classes of algorithms
  - Max Margin – Max the margin between optimal policy and value function
  - Bayesian – Use prior knowledge to model the optimal reward
  - Maximum Entropy
- Extensions
  - Multi-Agent Inverse Reinforcement Learning
  - Suboptimal policy demonstrations
  - Feedback-Types

Literature: Adams et al. (2022)

Topic 14

Neural Equilbirum Solvers

- Finding NEs and (C)CEs for normal-form games is hard
- Existing algorithms either may iterate large parts of the joint-action space or are iterative – which fail to converge in some cases
- E.g., meta-learning solvers solve for certain solution concepts in an inner loop, so that fast and constant time calculations to an approximate solution are sufficient
- This work trains a neural network that outputs approximate solutions with a parametrized objective
- „Turbocharging Solution Concepts: Solving NEs, CEs and CCEs with Neural Equilibrium Solvers", Marris et al., 2012 in Advances in neural information processing systems, 35

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Matching of Remaining Topics

Topics ‹› Students

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Q&A

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