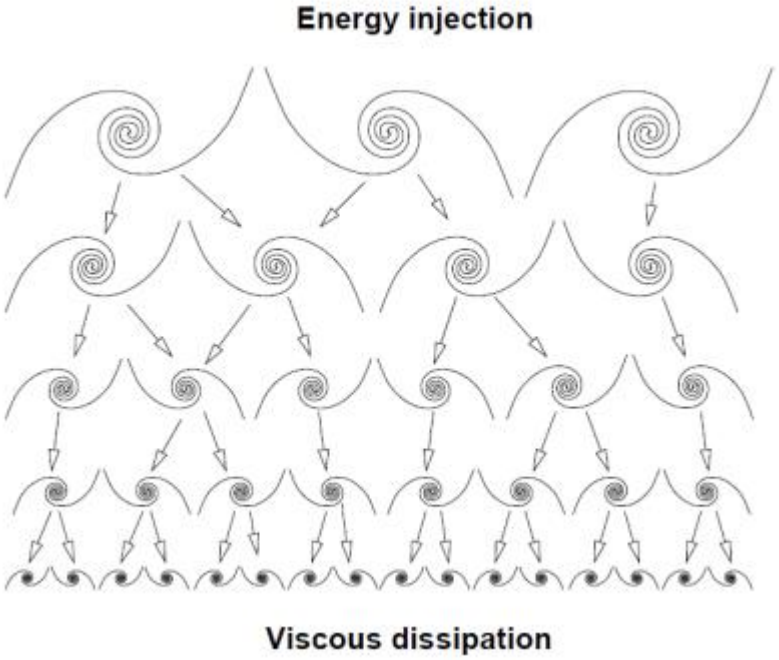
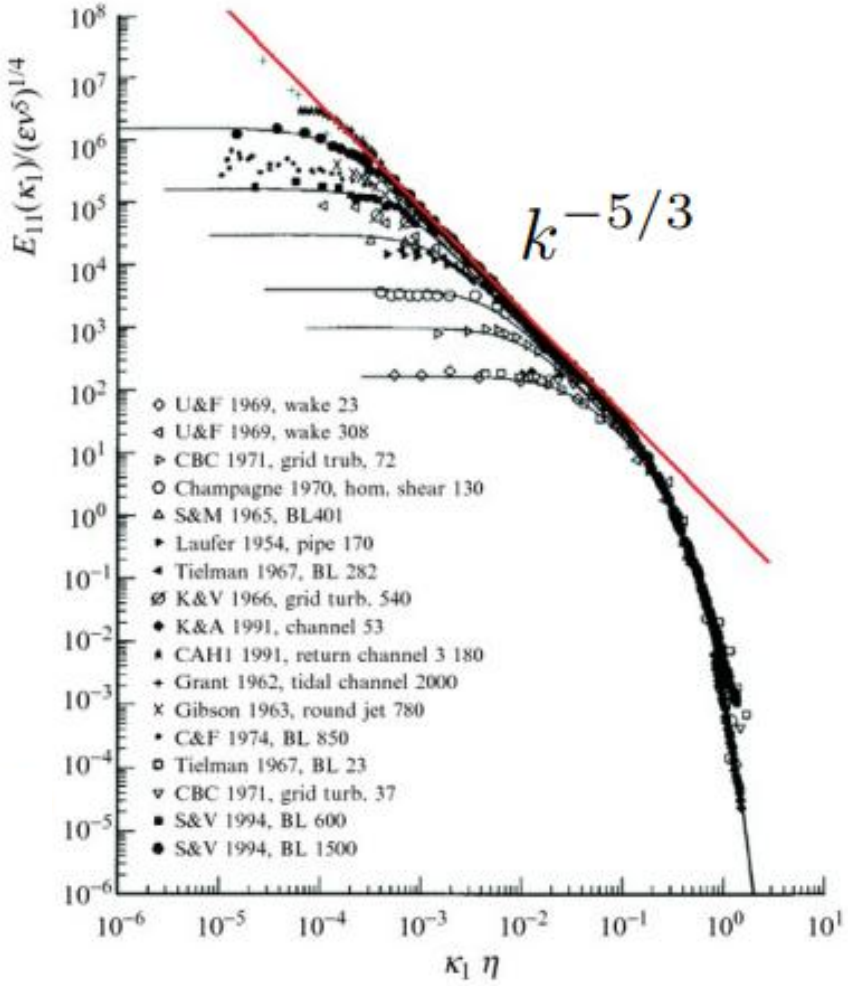
Energy Cascade in 3D Turbulence

$$E_k \propto \epsilon^{2/3} k^{-5/3}$$

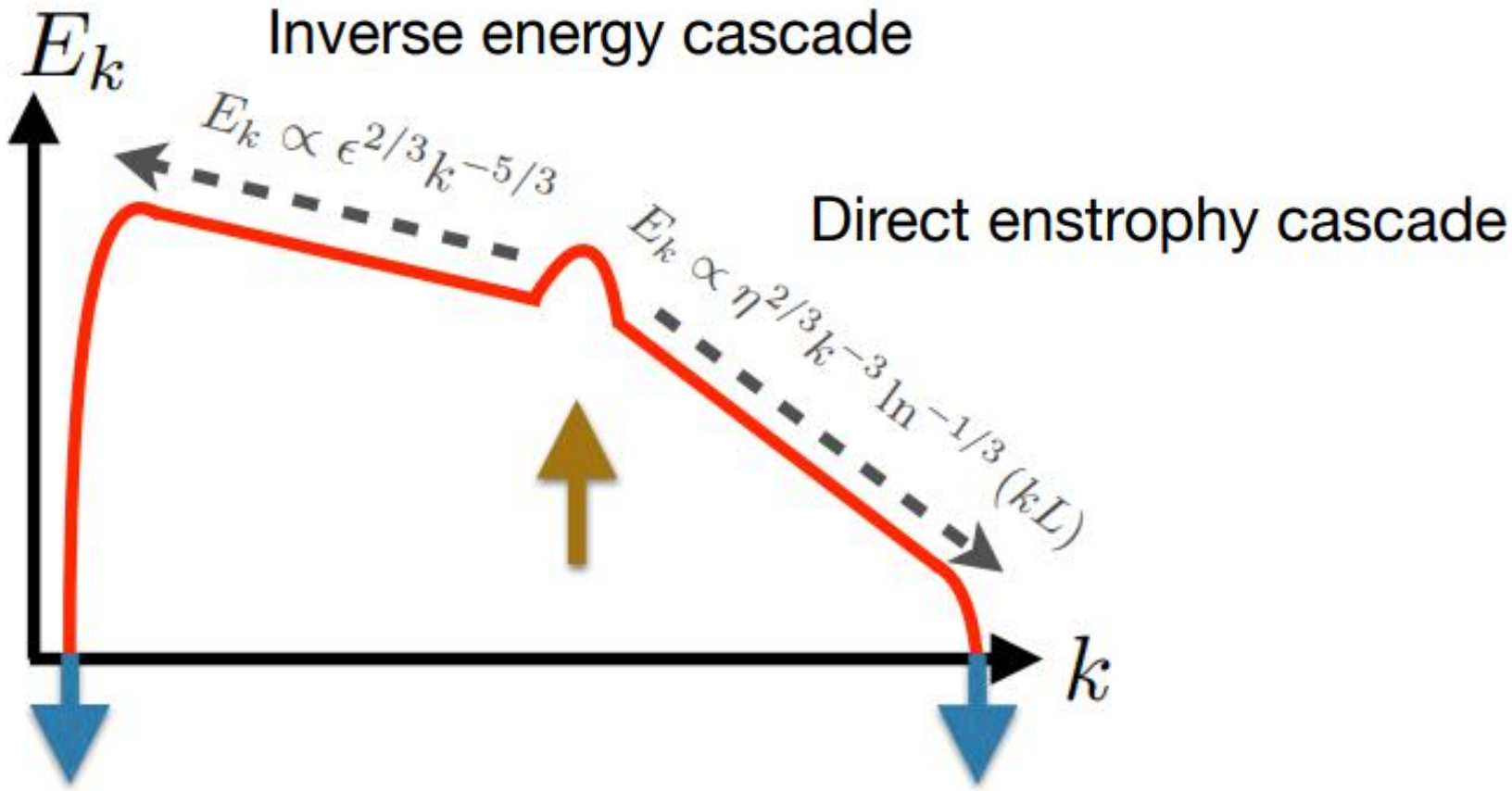




Energy Cascade in 2D Turbulence

$$E_k \propto \epsilon^{2/3} k^{-5/3}$$

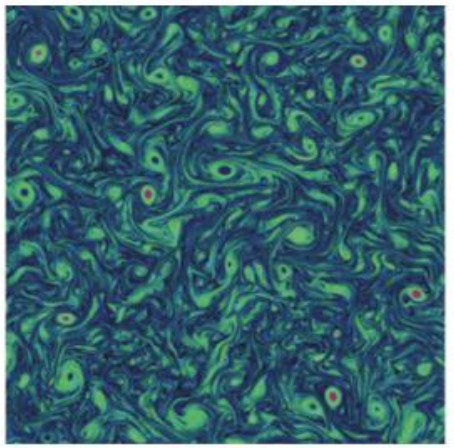
$$E_k \propto \eta^{2/3} k^{-3}$$

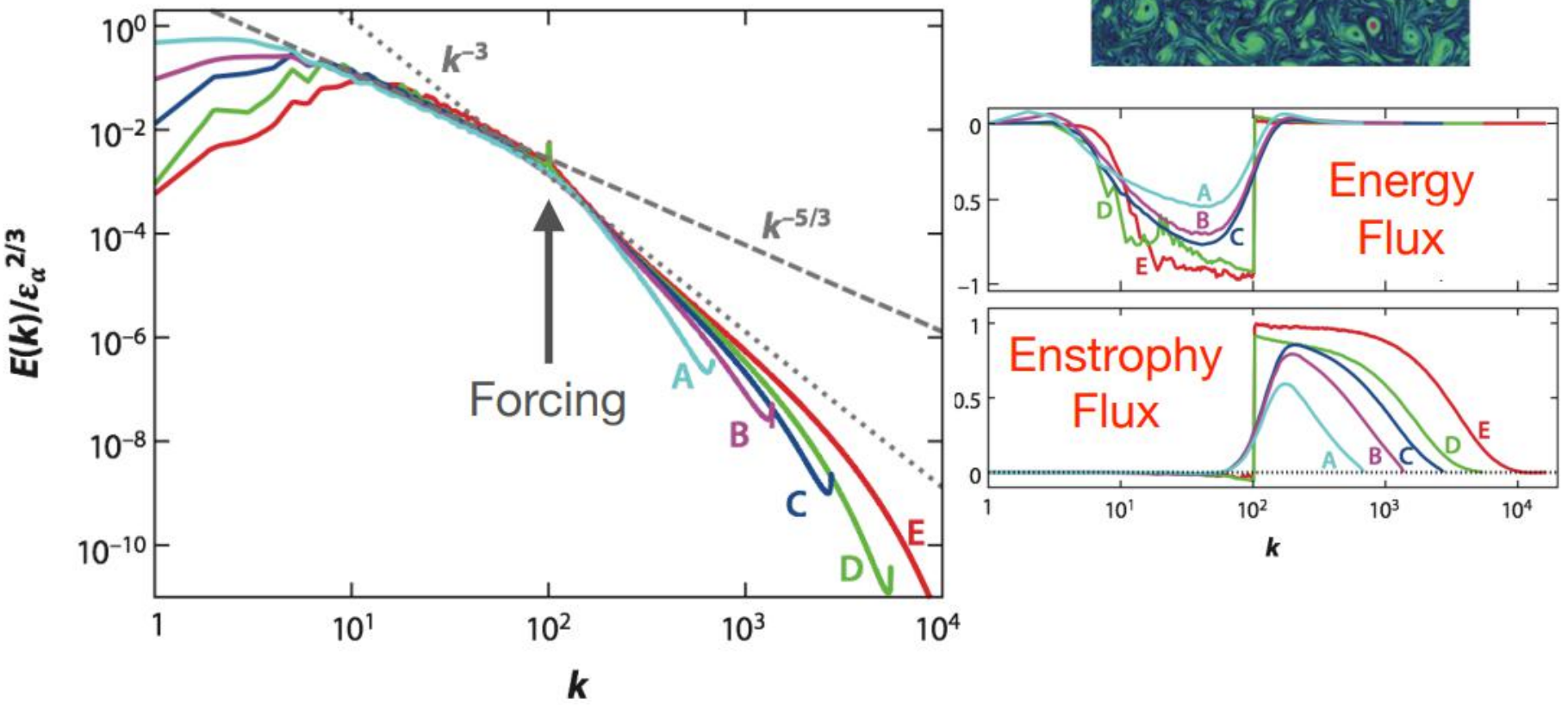


Energy Cascade in Kolmogorov Flow

2D Navier-Stokes with linear friction

$$\frac{\partial \omega}{\partial t} + \mathbf{v} \cdot \nabla \omega = \nu \nabla^2 \omega - \alpha \omega + f_\omega$$





Conclusion

Does 2D turbulence have the same energy cascade as 3D turbulence?

No. The energy spectrum for 2D turbulence has **two parts** with different slopes corresponding to inverse enerygy cascade and entropy cascade. Specifically for kolmogorov flow, the scale of the **forcing** determins the threshold of the cascade transform.

1. Robert H. Kraichnan; **Inertial Ranges in Two-Dimensional Turbulence**. Phys. Fluids 1 July 1967; 10 (7): 1417–1423. https://doi.org/10.1063/1.1762301

2. Boffetta, G. and Musacchio, S.; **Evidence for the double cascade scenario in two-dimensional turbulence**. Phys. Rev. E 1 July 2010; 82 (1): 016307–016312. https://link.aps.org/doi/10.1103/PhysRevE.82.016307

3. Jason Laurie, **An Introduction to 2D Turbulence.** Waves, coherent structures, and turbulence workshop,UEA, 30th October 2019, https://davideproment.github.io/WCST2019/slides/Laurie.pdf

About Seminar

- The final grades will be uploaded around the **mid of February**.
- We are open for guided research, thesis, Ph.D position. If you are interested, please send your C.V and transcript to Prof.Nils Thuerey.

Any questions?

Thanks for joining the seminar!

Merry Christmas and a Happy New Year!