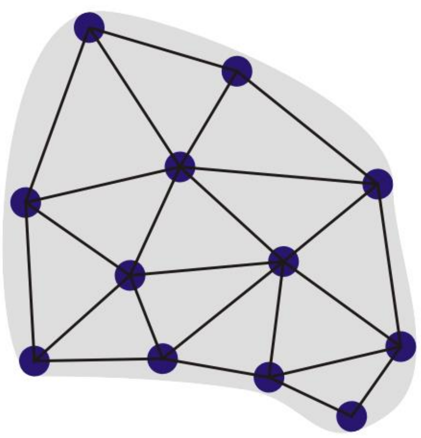
Outline

follows the structure of the report

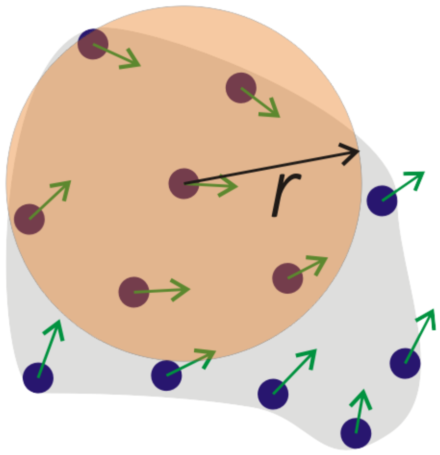
- Introduction
- Mesh-based Modeling of Cuts
- Finite Element Simulation of Virtual Cutting
- Numerical Solvers
- Meshfree Methods
- Summary & Application Study
- Discussion & Conclusion

Meshfree Methods

- Model objects as a set of interacting nodes which carry properties, e.g., mass, density, velocity, …
  - Introduced to computer graphics by Desbrun & Cani 1995
  - Re-formulated with continuum mechanics by Müller et al. 2004
- No explicit connectivity information
- Maintain node adjacency implicitly by an influence radius



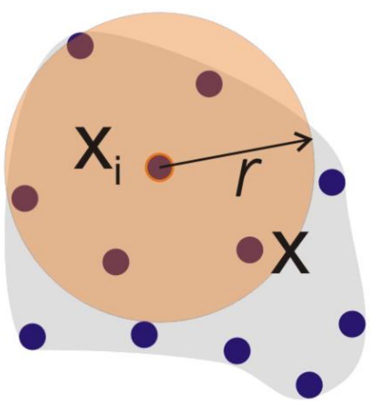
Mesh-based discretization



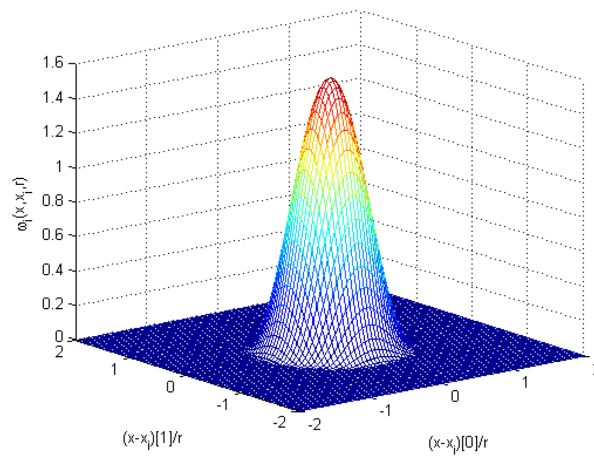
Meshfree discretization

Influence Radius & Weighting Kernel

- Moving Least Squares Approximation [Lancaster & Salkauskas 1981]
- Interpolation: $u(x) = \sum_i \phi_i(x)\, u_i$, for all $i \in \{i \mid d(x, x_i) \le r\}$
  - r: Influence radius
- Shape function: $\phi_i(x) = \omega_i(x, x_i, r) p^T(x)[M(x)]^{-1} p(x_i)$
  - Polynomial basis of order n: $p(x) = [x^0\ x^1 \dots x^n]^T$
  - Moment matrix: $M(x) = \sum_i \omega_i(x, x_i, r_i) p(x_i) p^T(x_i)$



Influence radius: r

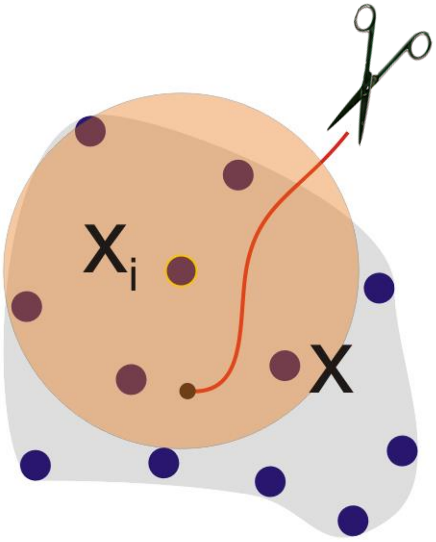


Weighting kernel: $\omega_i(x, x_i, r)$

$$\omega_i(x, x_i, r) = \begin{cases} \frac{315}{64\pi r^3}(r^2 - d^2(x, x_i))^3 & d(x, x_i) \le r \\ 0 & d(x, x_i) > r \end{cases}$$

Modeling Discontinuity

- Weighting kernel: $\omega_i(x, x_i, r) = \begin{cases} \text{nonzero} & d(x, x_i) \le r \\ 0 & d(x, x_i) > r \end{cases}$
  - Imply x_i and x are (implicitly) connected if the distance is smaller than the influence radius
- Modeling discontinuity by modifying the weighting kernel

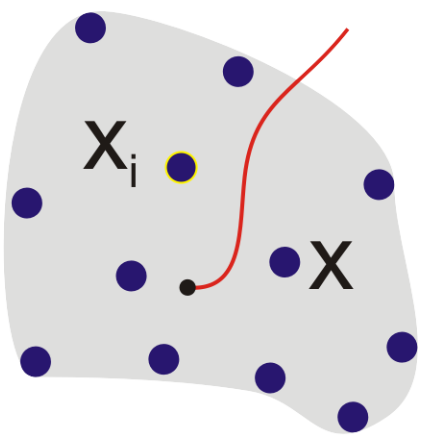


Cutting a meshfree object

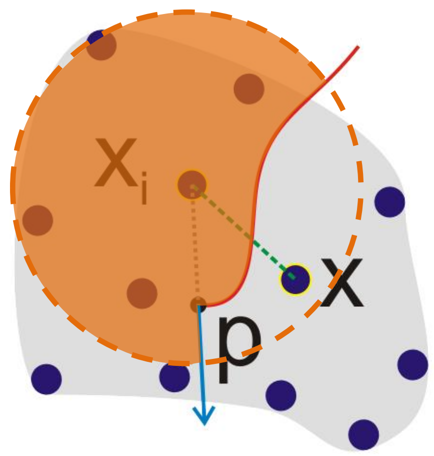
Modeling Discontinuity

- Visibility criterion: assign zero to $\omega_i(x, x_i, r)$, if x is invisible from x_i, i.e., $\overrightarrow{x x_i}$ intersects the cutting path [Belytschko et al. 1994]
- Weighting kernel:

$$\omega_i(x, x_i, r) = \begin{cases} \text{nonzero} & d(x, x_i) \le r \ \wedge \ x \text{ is visible} \\ 0 & d(x, x_i) > r \ \vee \ x \text{ is invisible} \end{cases}$$



Cutting a meshfree object

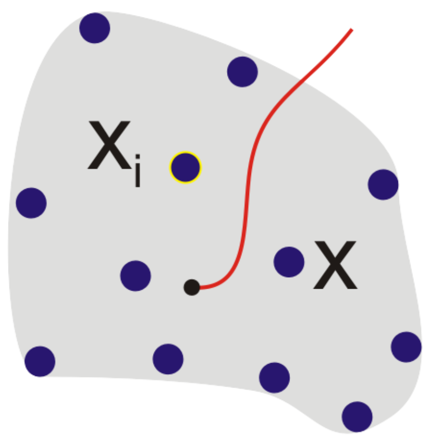


Visibility criterion

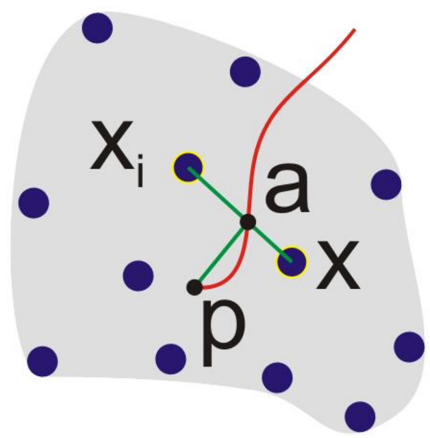
Modeling Discontinuity

- Transparency method: add to the Euclidean distance $d(x, x_i)$ a factor that depends on the distance $d(p, a)$ [Organ et al. 1996]
- E.g., $\omega_i(x, x_i, r) = \begin{cases} \frac{315}{64\pi r^3}\left(r^2 - d^2(x, x_i)\right)^3 & d(x, x_i) \leq r \\ 0 & d(x, x_i) > r \end{cases}$

$\left(d(x, x_i) + d(p, a)\right)^2$



Cutting a meshfree object



Transparency method

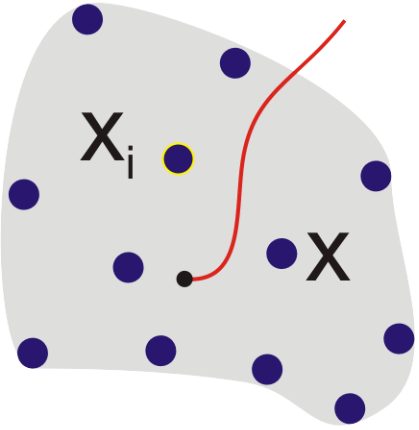
p: the discontinuity tip
a: the intersection

Modeling Discontinuity

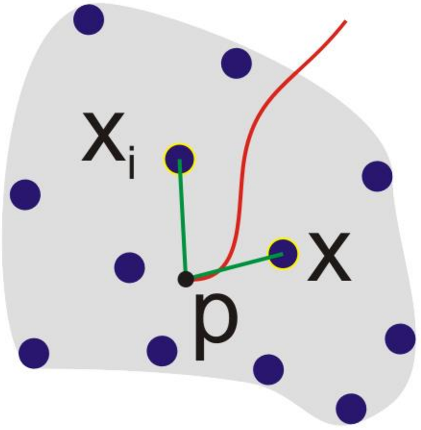
- Diffraction method: replace Euclidean distance $d(x, x_i)$ with the distance $d(p, x)$ and $d(p, x_i)$ [Organ et al. 1996]

- E.g., $\omega_i(x, x_i, r) = \begin{cases} \frac{315}{64\pi r^3}\left(r^2 - d^2(x, x_i)\right)^3 & d(x, x_i) \le r \\ 0 & d(x, x_i) > r \end{cases}$

$\left(d(p, x) + d(p, x_i)\right)^2$



Cutting a meshfree object



Diffraction method

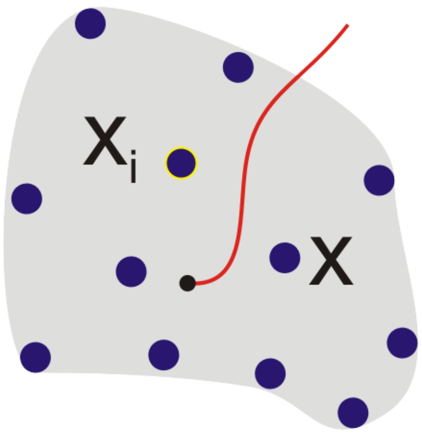
p: the discontinuity tip

In 3D the position of p is not well defined

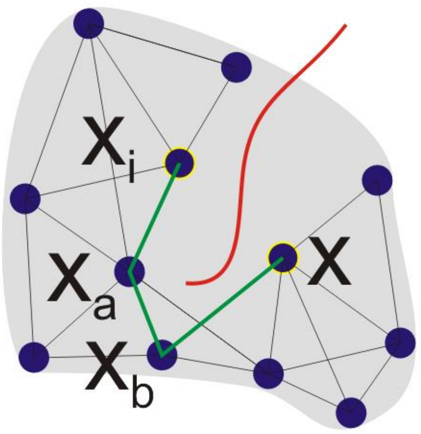
Modeling Discontinuity

- Graph-based diffraction method: replace Euclidean distance $d(x, x_i)$ with the minimum distance $x_i \to x$ in a graph
  [Steinemann et al. 2006]
- E.g., $\omega_i(x, x_i, r) = \begin{cases} \frac{315}{64\pi r^3}\left(r^2 - d^2(x, x_i)\right)^3 & d(x, x_i) \leq r \\ 0 & d(x, x_i) > r \end{cases}$

$d^2(x_i \to x_a \to x_b \to x)$



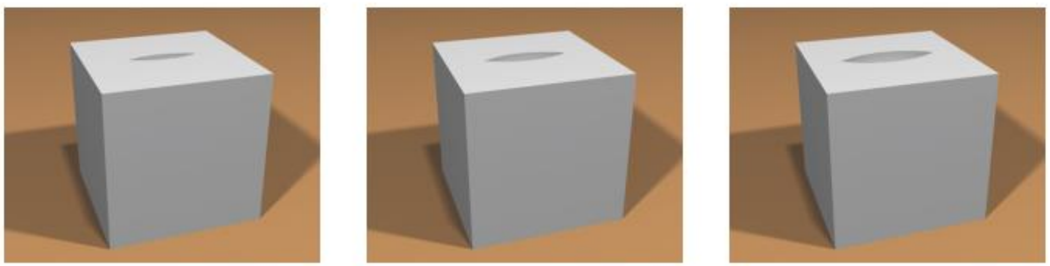
Cutting a meshfree object

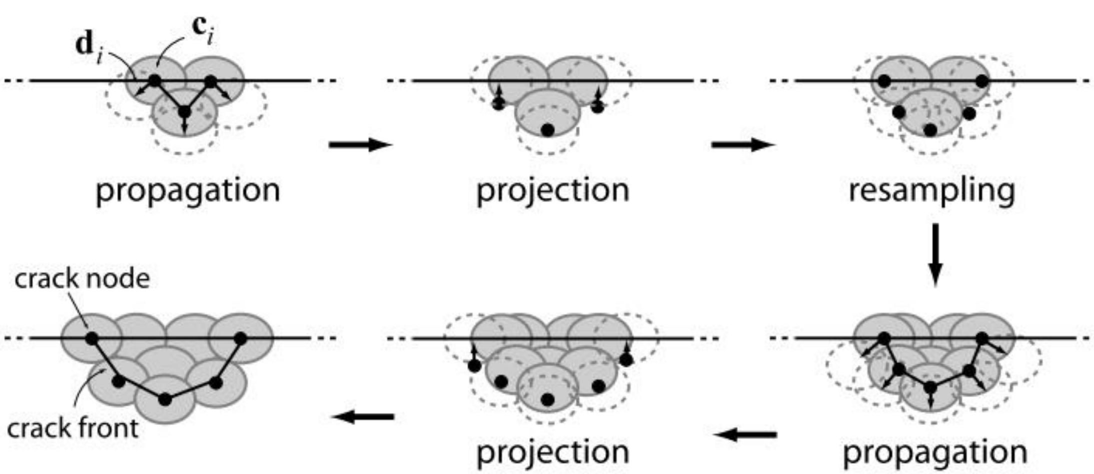


Graph-based diffraction method

Generating New Surface due to Cuts

- Crack surface propagation [Pauly et al. 2005]
  - Represent surface by means of elliptical splats (surfels)
  - Propagate crack front and create additional surfels when necessary



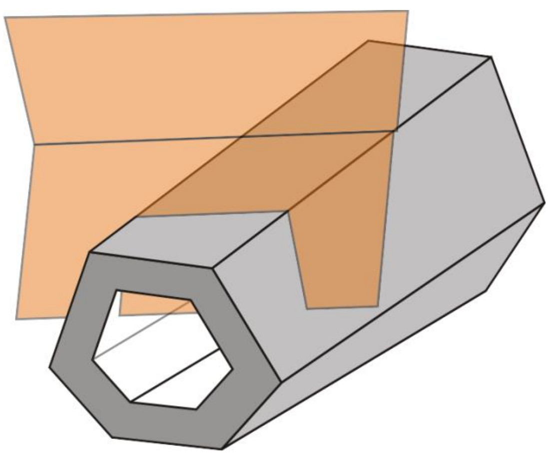


Meshless fracture animation

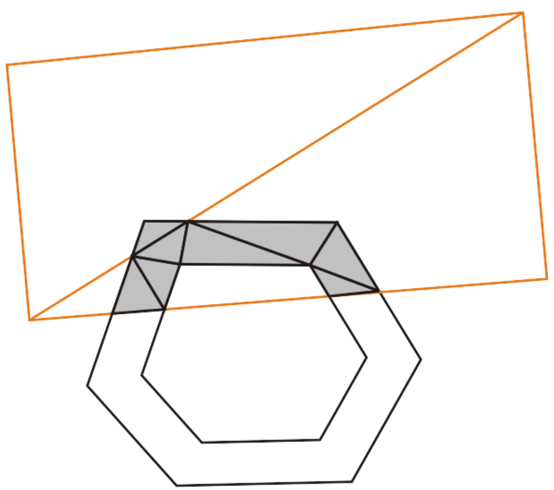
[Pauly et al. 2005]

Generating New Surface due to Cuts

- Explicit cutting surface modeling [Steinemann et al. 2006]
  - Represent cutting surface as a triangle mesh
  - Trim this surface by the initial, triangulated surface of the object



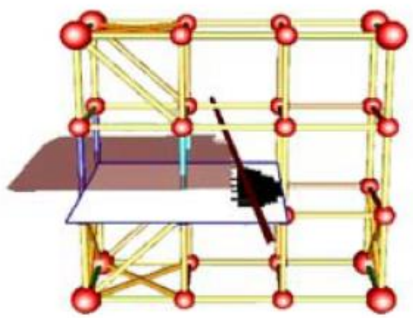
Cutting configuration



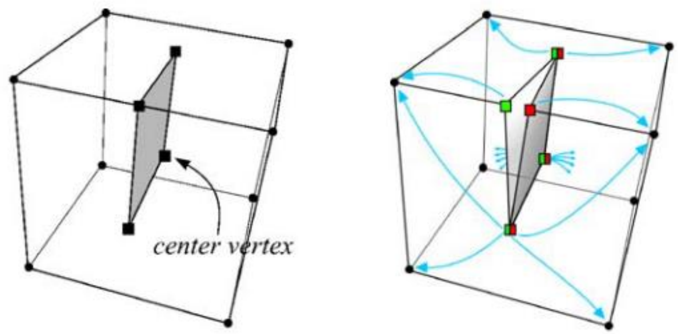
Trimming and triangulation

Generating New Surface due to Cuts

- Surface reconstruction based on a regular hexahedral grid [Pietroni et al. 2009]
  - Deformable body is embedded into a regular hexahedral grid
  - Separate edges of grid cells by cutting tool
  - Reconstruct a triangle mesh from the disconnected edges, using intersection points and normal at these points



Separating of edges



Reconstruction of a triangle mesh

[Pietroni et al 2009]

Discussion on Meshfree Methods

- Advantages:
  - No re-meshing required (volume and surface)
- Disadvantages:
  - Handling of essential boundary conditions is difficult
  - Neighborhood among nodes must be determined during run-time
  - Inversion of the moment matrices is expensive

- Explicit connectivity can still be advantageous …
  - A graph representation can be used to efficiently determine neighborhood [Steinemann et al. 2006]
  - A regular hexahedral grid can be used to contour the surface [Pietroni et al. 2009]