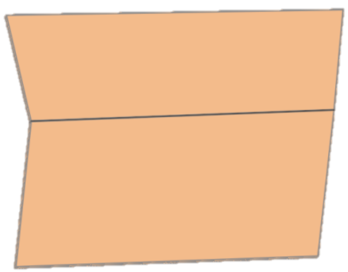
Outline

follows the structure of the report

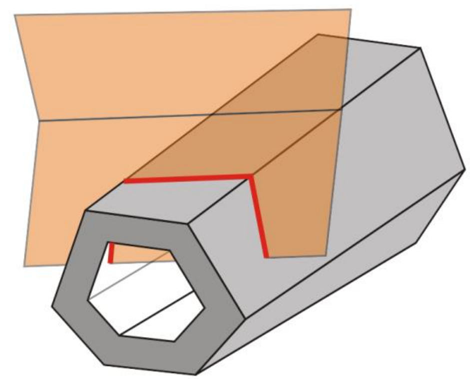
- Introduction
- Mesh-based Modeling of Cuts
  - Modeling of the Cutting Process
  - Tetrahedral Meshes
  - Hexahedral Meshes
  - Polyhedral Meshes
  - Discussion on Discretizations
- Finite Element Simulation for Virtual Cutting
- Numerical Solvers
- Meshfree Methods
- Summary & Application Study
- Discussion & Conclusion

Modeling of the Cutting Process

- Detect intersections between the volumetric mesh **(**the deformable object**)** and a surface mesh (cutting surface)
  - Edge-face test



Cutting surface mesh

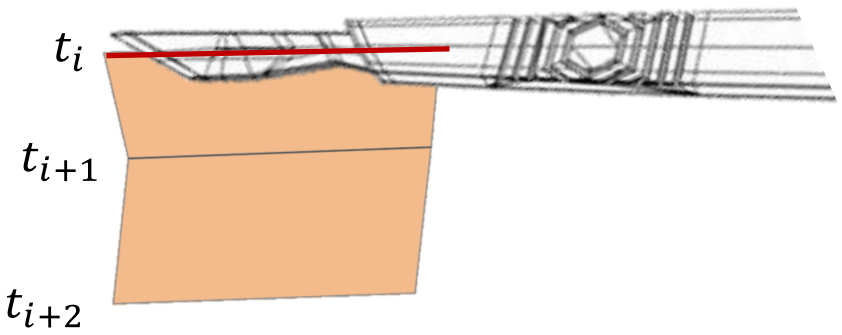


Object volumetric mesh

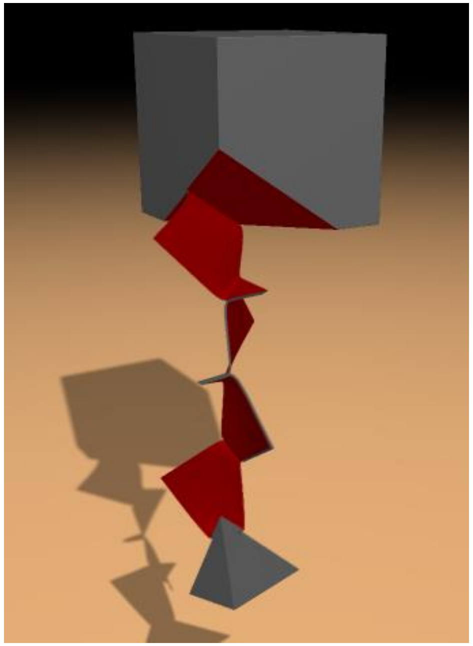
- Acceleration techniques
  - Bounding volume hierarchies
  - Breadth-first traversal of the volumetric mesh

Modeling of the Cutting Process

- Cutting surface generation
  - Swept surface of the cutting blade (interactive simulation)
  - Predefined cutting patterns (offline simulation)



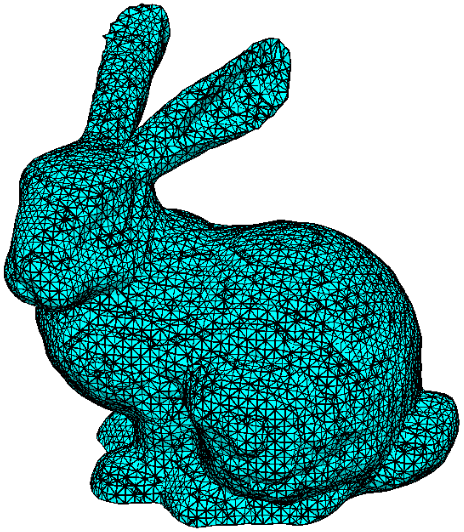
Swept surface



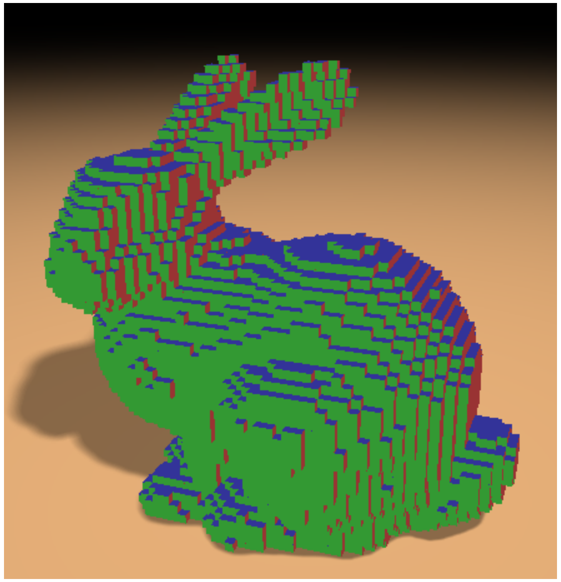
Cutting using a predefined pattern

Spatial Discretizations

- 2D: triangles, quadrangles, polygons
- 3D: tetrahedra, hexahedra, polyhedra



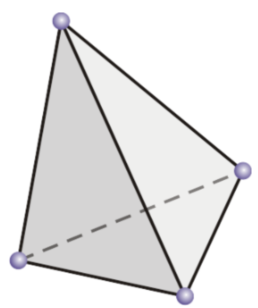
Tetrahedralized bunny model



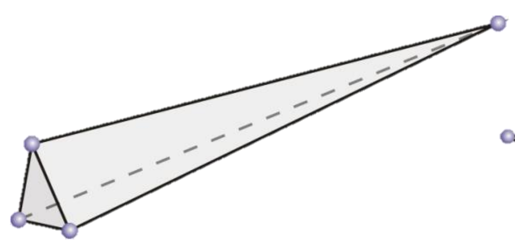
Hexahedralized bunny model

Tetrahedral Meshes

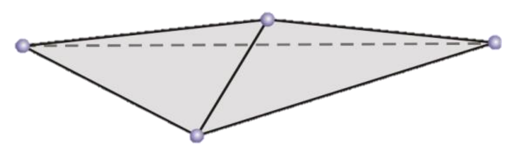
- Widely applied in computer graphics & engineering
- An initial tetrahedral discretization of the simulation domain can be generated
  - from surface meshes, medical images, level sets, et al.
  - by TetGen, LBIE-Mesher, et al.
- Challenge: avoiding ill-shaped meshes
  - Ill-shaped meshes lead to numerical instabilities
  - Mesh quality is ensured in the non-trivial initialization



Good-shaped



Ill-shaped, needle

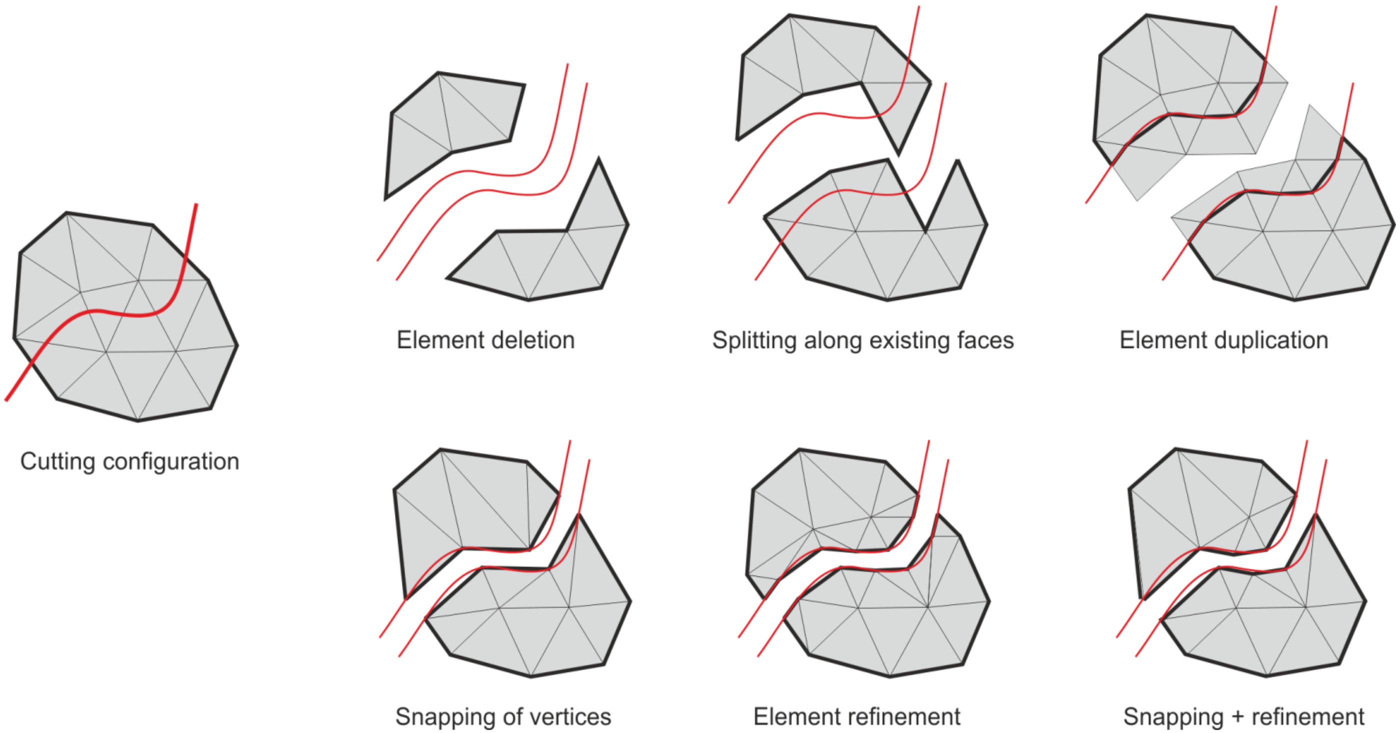


Ill-shaped, sliver

…

Tetrahedral Meshes

- Many techniques to model cuts into tetrahedral meshes

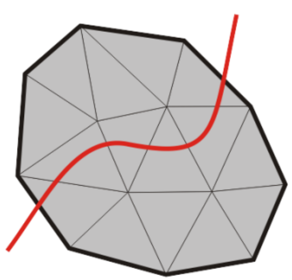


Techniques for modeling cuts in a tetrahedral mesh (a triangle mesh in 2D)

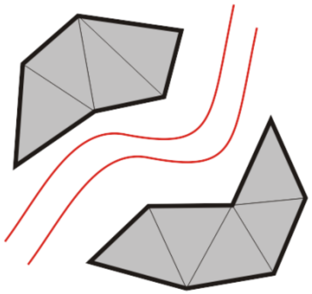
Cut Modeling without Creating New Elements

- Element deletion
  - Remove meshes that are touched by a cutting tool

➢ Simple, but result in a jagged surface and a loss of volume



Cutting configuration

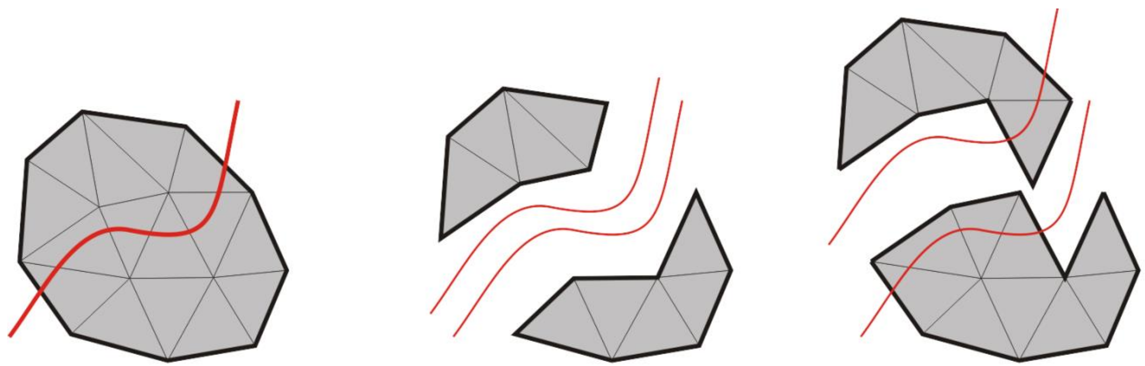


Element deletion

Cut Modeling without Creating New Elements

- Element deletion
- Splitting along existing faces

➢ Simple, but result in a jagged surface ~~and a loss of volume~~



Cutting configuration

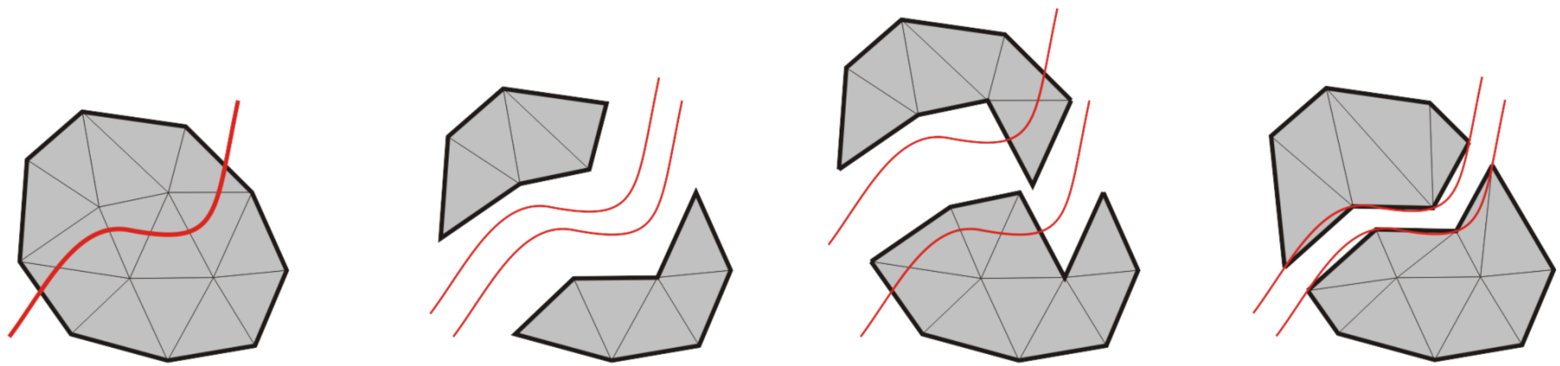
Element deletion

Splitting along existing faces

Cut Modeling without Creating New Elements

- Element deletion
- Splitting along existing faces
- Snapping of vertices
  - Snap vertices onto the cutting surface, i.e., positions altered
  - Then, split along faces

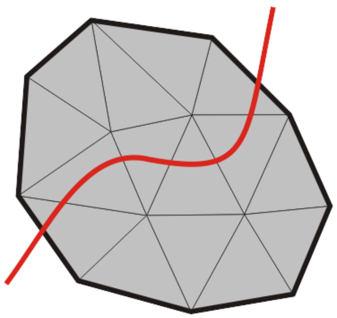
➢ Partially alleviate the jagged surface, but mesh quality cannot be ensured



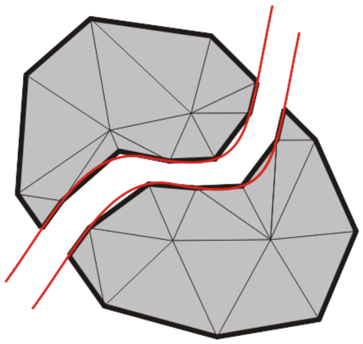
Cutting configuration | Element deletion | Splitting along existing faces | Snapping of vertices

Cut Modeling by Element Refinement

- Motivation: to accurately model a cut
- Solution: refine meshes along the cut
  - Split edges at the exact intersections
  - Create new, smaller meshes



Cutting configuration

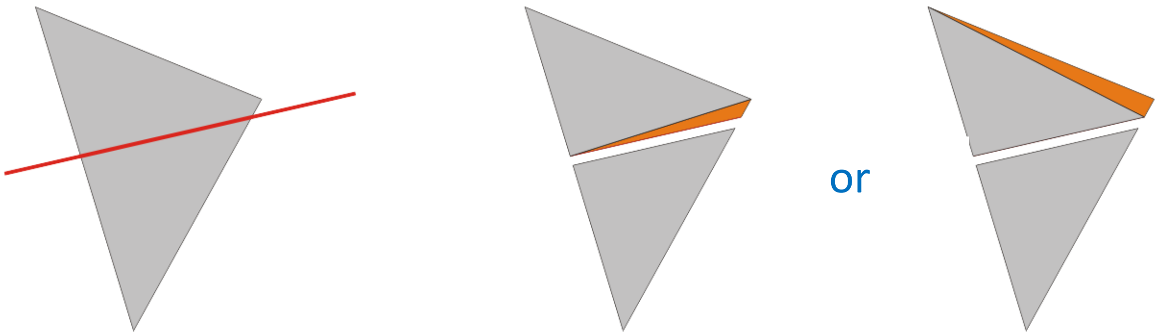


Element refinement

Cut Modeling by Element Refinement

- Motivation: to accurately model a cut
- Solution: refine meshes along the cut
  - Split edges at the exact intersections
  - Create new, smaller meshes

➢ Geometrically accurate, but easily lead to ill-shaped meshes
  - If the intersection is close to an initial vertex

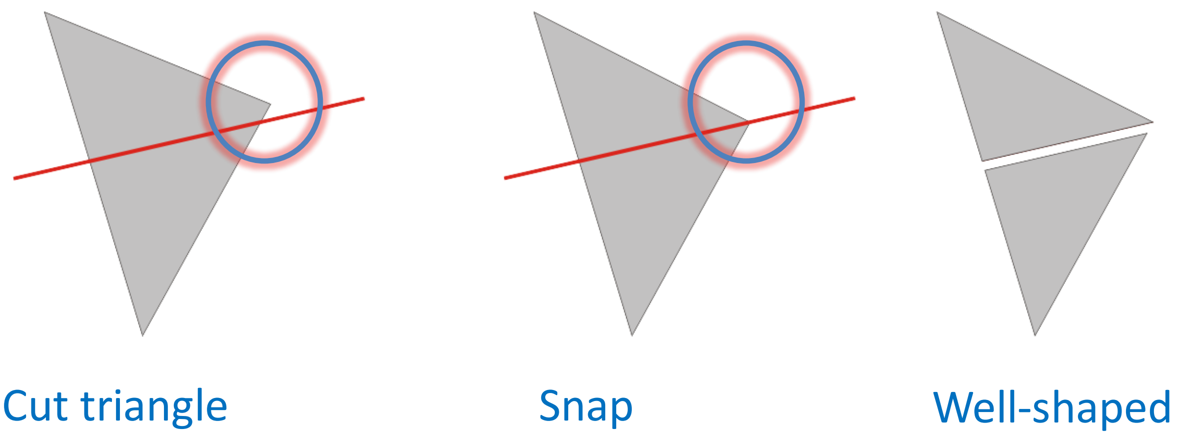


Cut triangle

Ill-shaped, needles

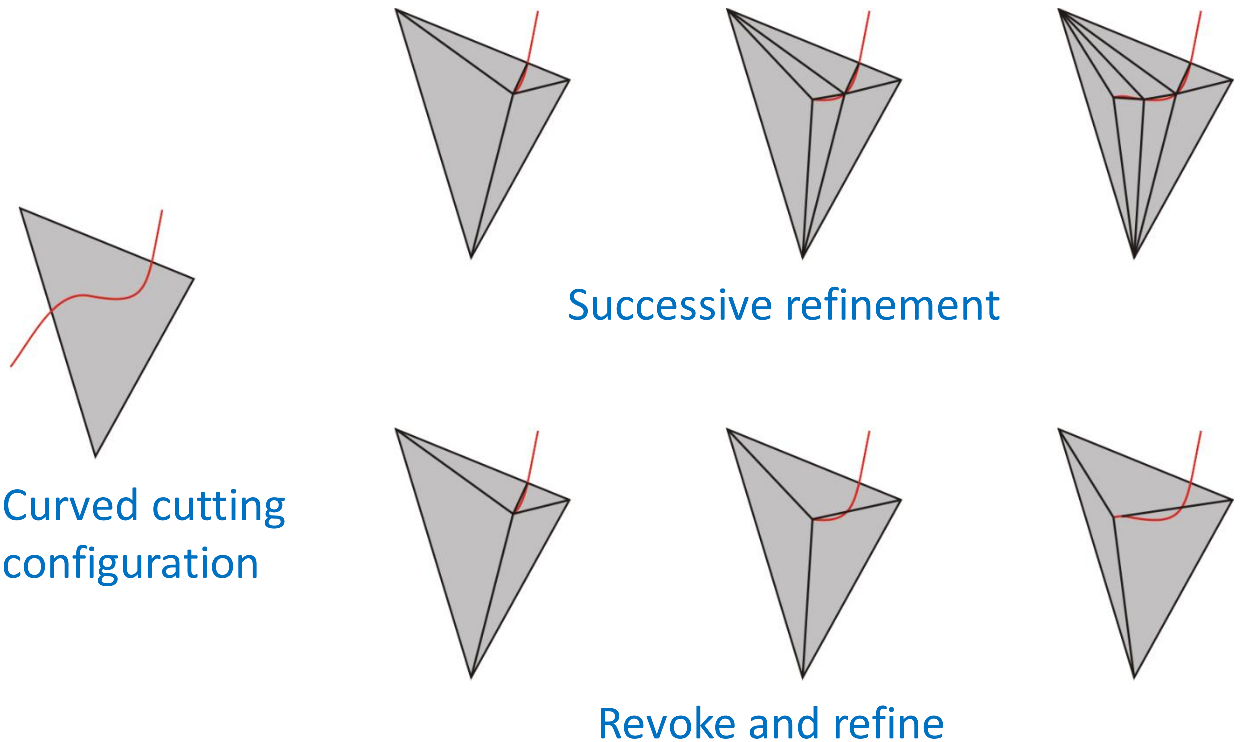
Cut Modeling by Element Refinement

- Motivation: to improve mesh quality
- Solution: a combination of snapping & refinement
  - Snap the vertex, if the intersection is close to it
  - Split the edge, otherwise



Cut Modeling by Element Refinement

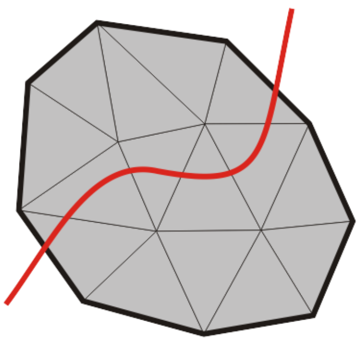
- Incremental, curved cutting path within one mesh
- Solutions:
  - Successive refinement
  - Revoke and refine



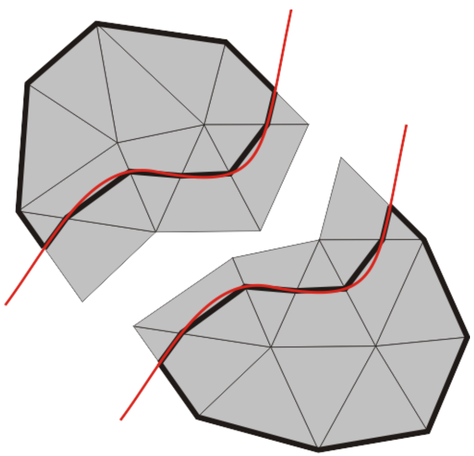
Cut Modeling by Element Duplication

- Motivation: to avoid ill-shaped elements

- Solution: duplicate the initial well-shaped elements
  - Create replicas of the elements that are cut
  - Embed material surfaces into a unique replica



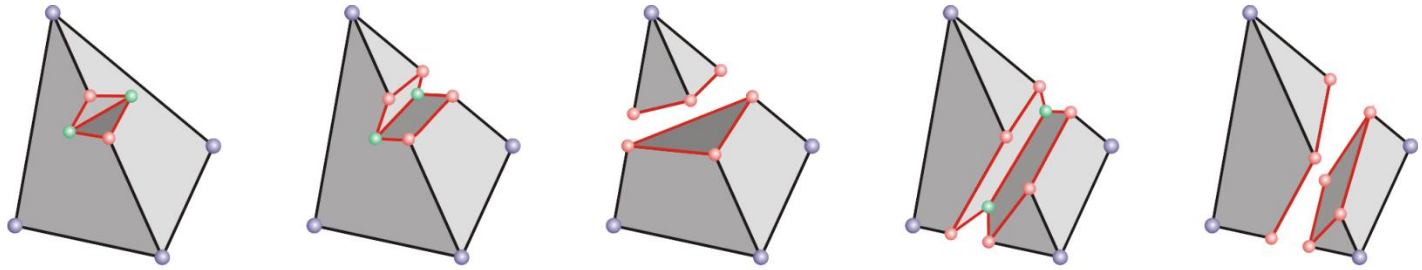
Cutting configuration



Element duplication

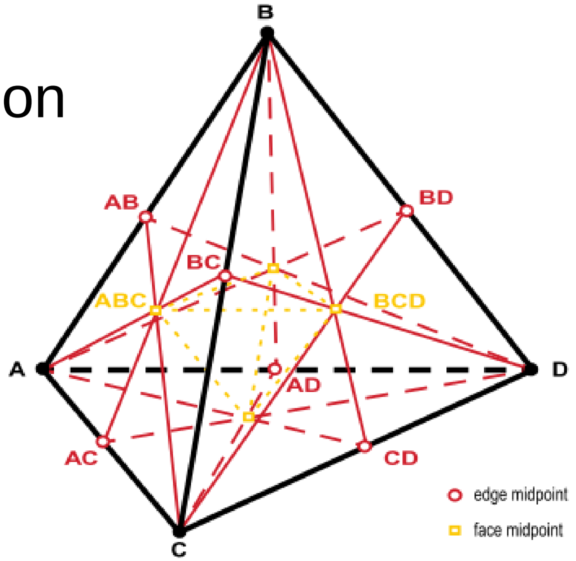
Tetrahedral decomposition

- Topological configurations of a cut tetrahedron



I (6) II (12) III a (4) III b (12) IV (3)

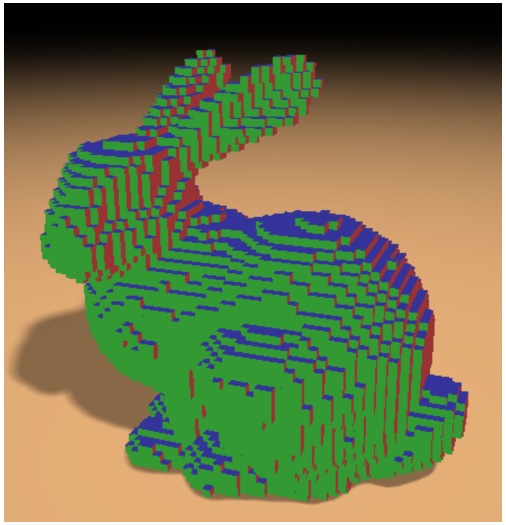
- Generic 1:17 tetrahedral decomposition
  - Add a vertex on each edge
  - Add a vertex on each triangle face
  - Exact placements decided by intersection tests



[Biielser et al. 1999]

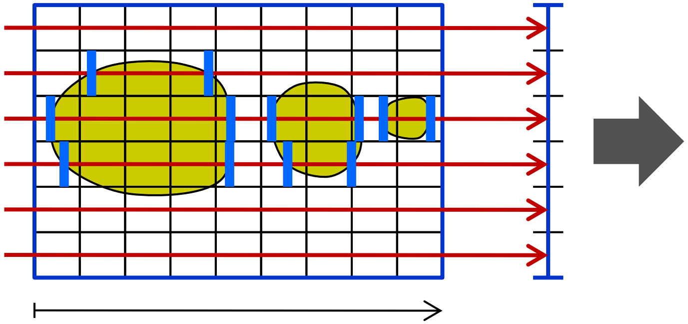
Hexahedral Meshes

- Each element has a regular shape
- No worry about numerical instabilities!



Hexahedralized bunny model

- Generated from
  - medical images
  - polygonal surfaces by voxelization

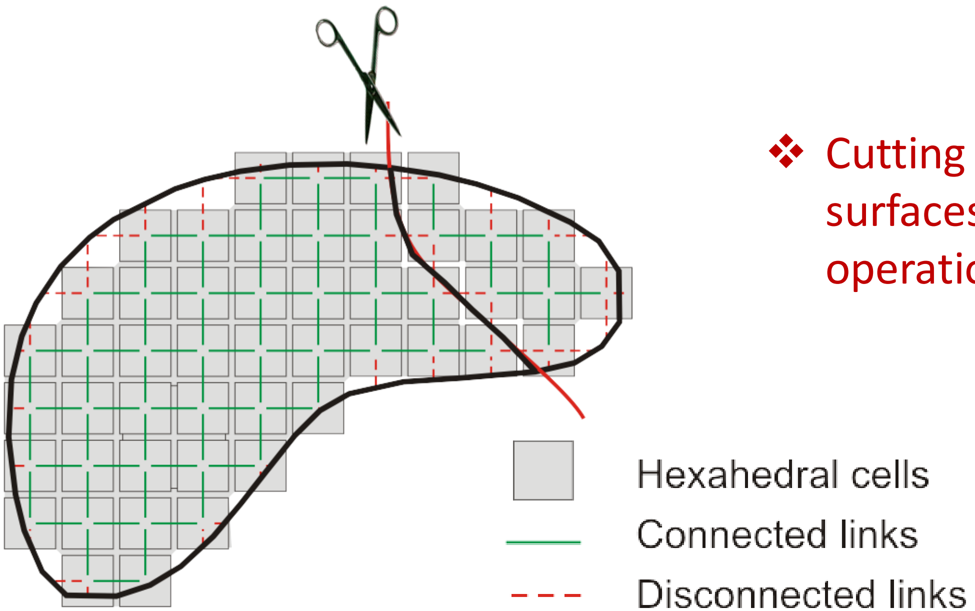


0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0
1	1	1	1	0	1	1	0	1
0	1	1	1	0	0	1	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

2D illustration of voxelization

Hexahedral Meshes - Volume Representation

- Linked volume
  - Decompose the object into a set of uniform hexadedra
  - Connect face-adjacent elements by links
  - Cutting: break the link between elements

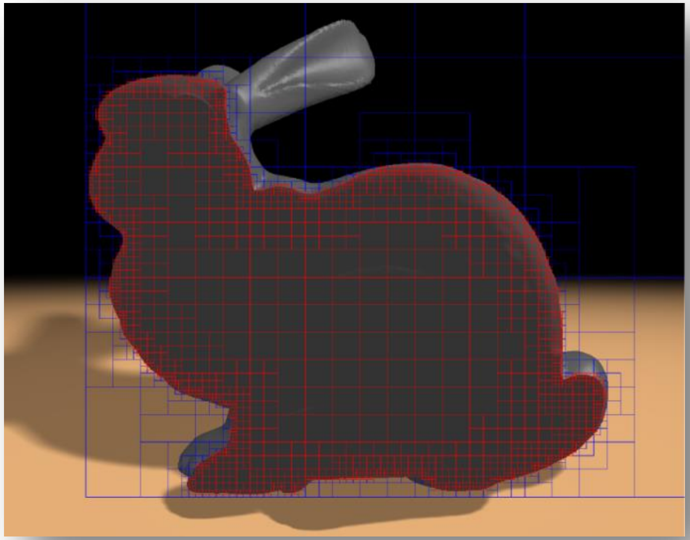


2D illustration of cutting on a linked volume

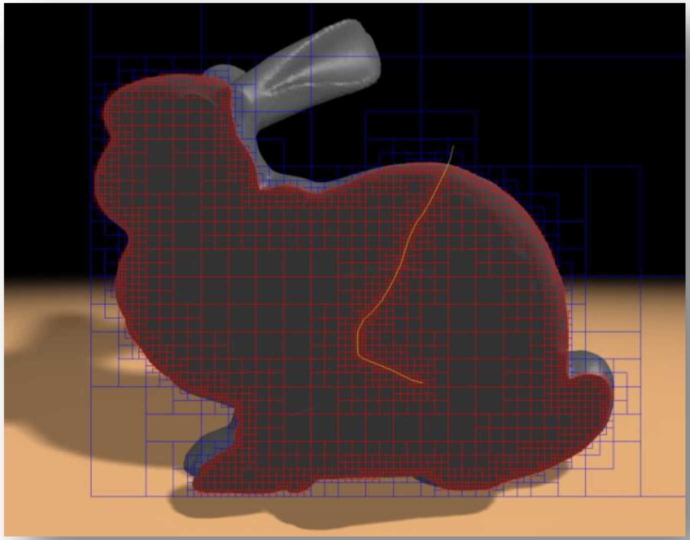
❖ Cutting surfaces and object boundary surfaces are both considered as cutting operations to break the links

Hexahedral Meshes - Volume Representation

- Adaptive linked octree
  - Cutting: refine local elements, then break links
  - Regular 1:8 hexahedral decomposition
    - Efficient
    - No ill-shaped elements



Initial octree



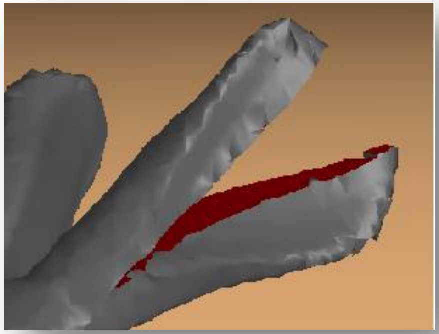
Refined octree

Hexahedral Meshes - Surface Representation

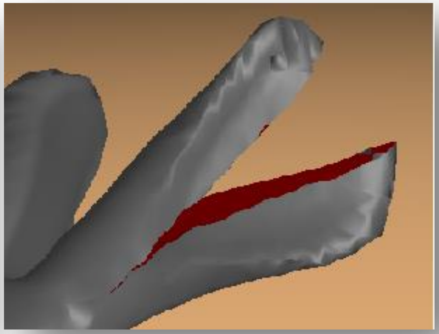
- Surface reconstruction methods
  - Marching cubes
  - Splitting cubes
  - Dual contouring

[Jeřábková et al. 2010]

Using marching cubes



Using splitting cubes

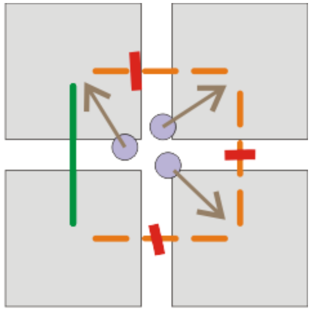


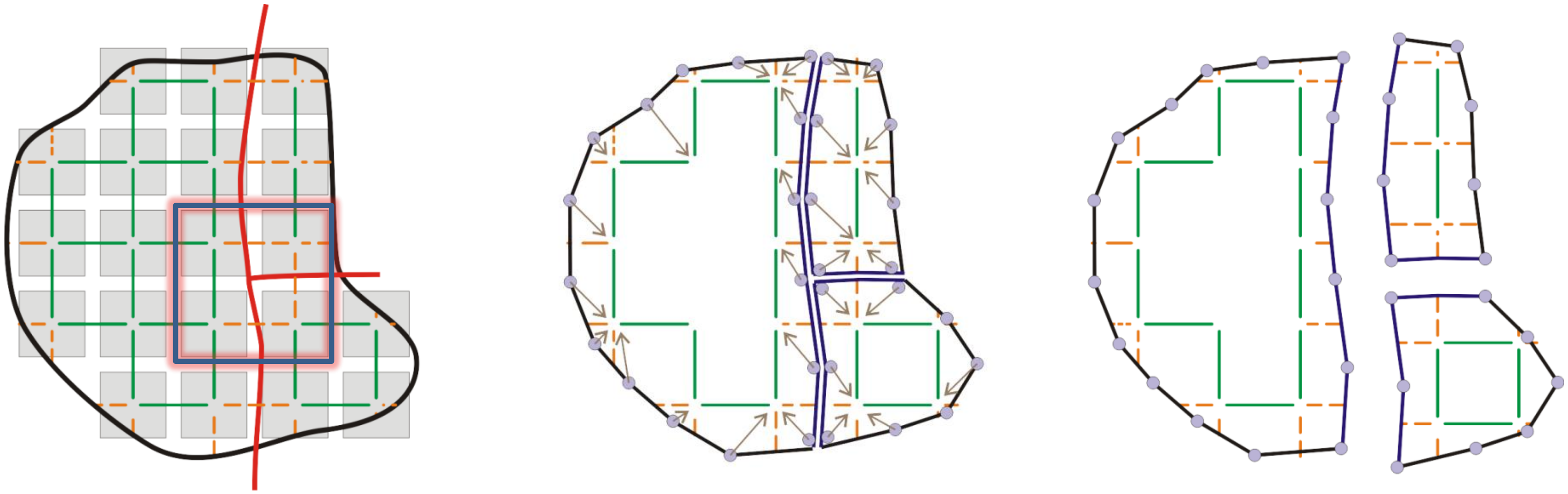
Using dual contouring

Surface reconstruction after cutting by different methods

Hexahedral Meshes - Surface Reconstruction

- Input: positions of intersection points & cutting normals
- Algorithm: (For each 2^3 block of elements)
  - Compute a surface vertex position which best matches all cuts
  - Duplicate the vertices as many times as the number of disconnected parts
  - Bind each replica to a volume element

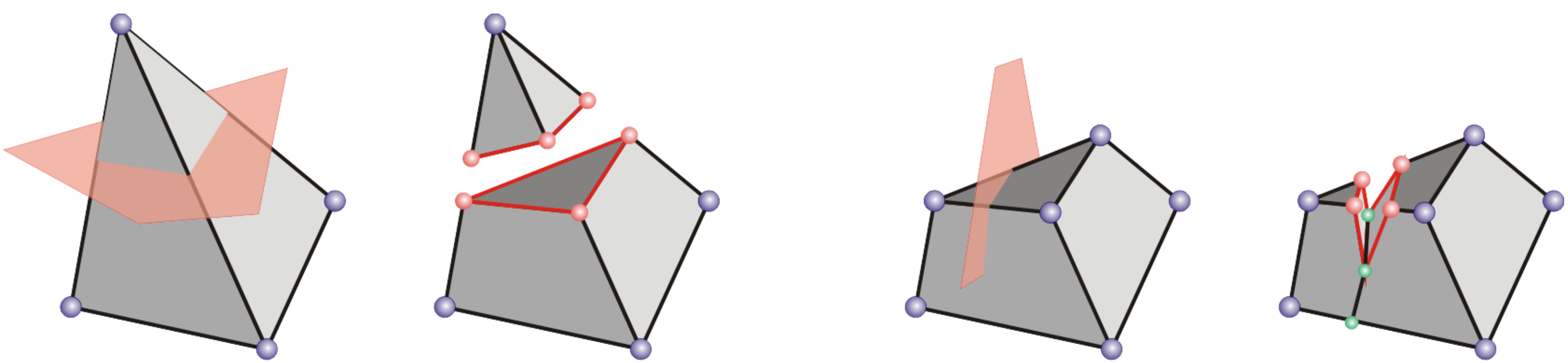




2D illustration of surface reconstruction

Polyhedral Meshes

- Flexible in representing shapes
  - Split the elements along a cutting plane
  - No further subdivision (e.g., tetrahedralization) is required
- Pros: no further subdivision is required
- Cons: ill-shaped elements needs to be avoided



A tetrahedron -> A small tetrahedron & A triangular prism

A tetrahedron -> Two small tetrahedra

Discussion on Discretizations

- Tetrahedral & polyhedral meshes
  - Pros: flexibility in shape modeling, directly renderable surfaces
  - Cons: ill-shaped elements
  - Methods:
    - element deletion, splitting along existing faces, element duplication, snapping of vertices, element refinement, snapping + refinement

- Hexahedral meshes
  - Pros: efficiency wrt. subdivision and solvers, stability
  - Cons: a separate surface is needed
  - Methods:
    - (adaptive) linked volume, surface reconstruction