Inverse Volume Rendering

Absorption + Emission, Automatic Differentiation

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Technische Universität München

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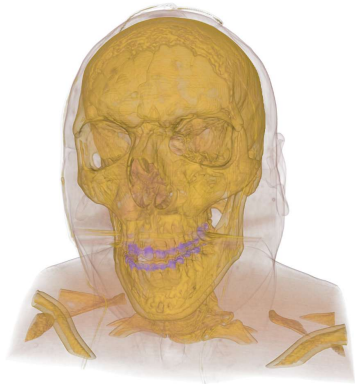
Overview

1. Models and Assumptions
   - 1.1. Absorption
   - 1.2. Emission
   - 1.3. Transfer Function



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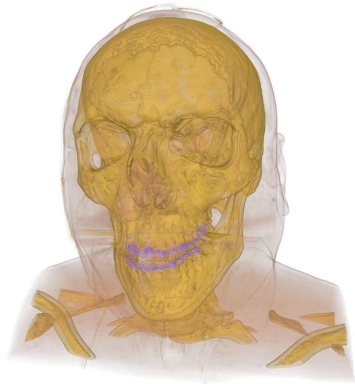
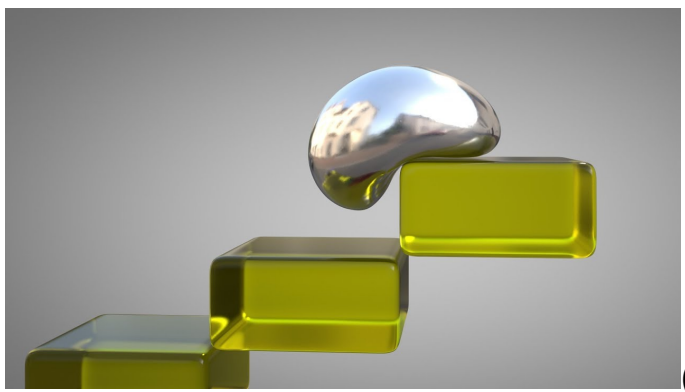
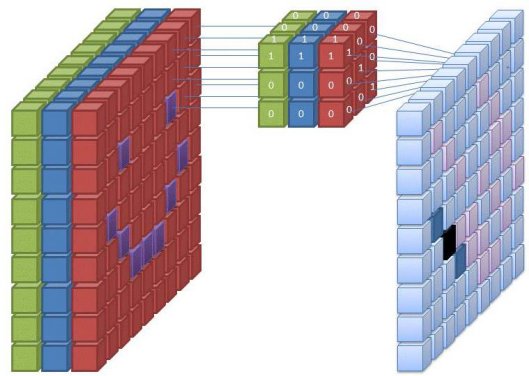


2. Automatic Differentiation
   - 2.1. Forward Mode
   - 2.2. Backward Mode

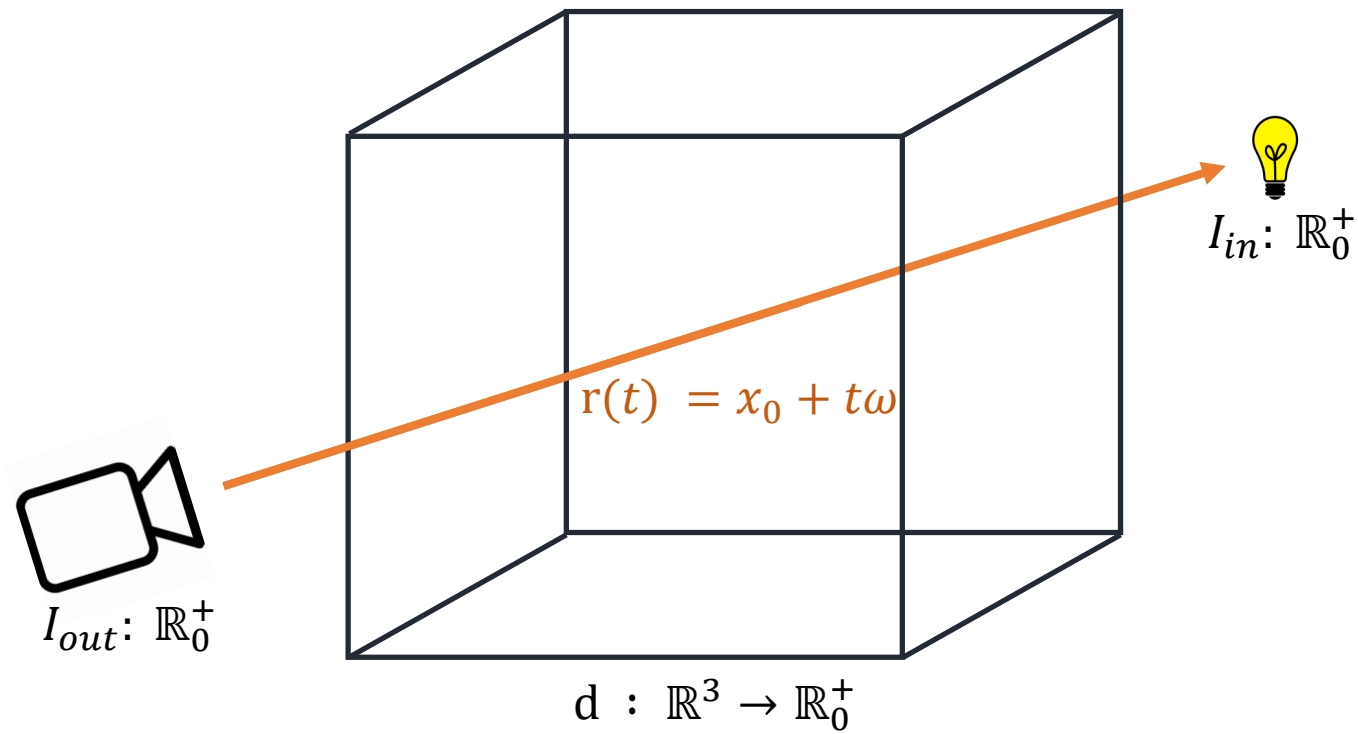

(1)


(2)

(1) https://www.youtube.com/watch?v=HwJyg5o3HGg
(2) https://commons.wikimedia.org/wiki/File:Convolutional_Neural_Network_with_Color_Image_Filter.gif

1. Models and Assumptions

1. Scalar Field $d : \mathbb{R}^3 \to \mathbb{R}_0^+$ of densities
2. Ray $r(t) = \boldsymbol{x}_0 + t\boldsymbol{\omega}$, $\boldsymbol{x}_0 \in \mathbb{R}^3, \boldsymbol{\omega} \in S^2$
3. Received Illumination $I_{\text{out}}(\boldsymbol{x}_0, \boldsymbol{\omega}) \in \mathbb{R}_0^+$



1.1 Only Absorption

Assume a medium that only absorbs incoming light, no emission or scattering.

$$I_{\text{out}}(\boldsymbol{x}_0, \boldsymbol{\omega}) = \int_0^{\infty} g(t) \exp\left(-\int_0^t \tau(v(r(t')))dt'\right) dt$$

$$\text{with } g(s) = c(s)\tau(s) + \int_{\Omega} p(\boldsymbol{\omega}, \boldsymbol{\omega}') I_{\text{out}}(r(t), \boldsymbol{\omega}')d\boldsymbol{\omega}'$$

Nelson Max. Optical Models for Direct Volume Rendering. *TVCG* 1995
Kajiya, von Herzen. Ray Tracing Volume Densities. *Computer Graphics* 1984

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Set $g(\infty) = I_{\text{in}}$ and zero everywhere else (only absorption).
Set $\tau(d) = d$

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Set $g(\infty) = I_{\text{in}}$ and zero everywhere else (only absorption).
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$$\Rightarrow \text{Transparency } T(s) = \exp\left(-\int_0^{s} d(r(t))dt\right)$$

$$\Rightarrow \text{Received Illumination } I_{\text{out}} = T(t_{\text{max}})I_{\text{in}}$$

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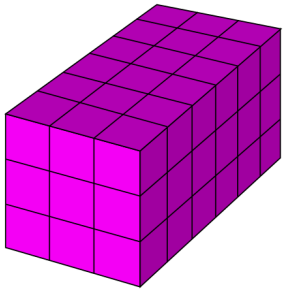
1.1 Only Absorption

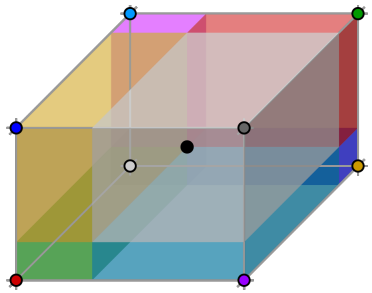
1.1 Only Absorption: Solutions

Densities on a grid v_i



(1)

Trilinear Interpolation $N_i(x)$



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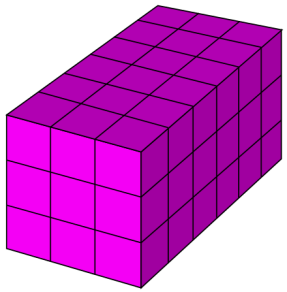

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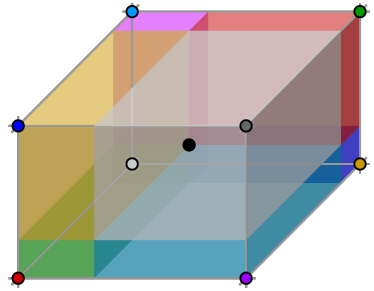

(2)

$$\Rightarrow \log I_{\text{out}} = \sum_{i \in \text{voxels}} \sum_{j=1}^{8} v_{ij} \int_{t_0^{(i)}}^{t_1^{(i)}} N_j(r(t))dt$$

Assemble into matrices for all rays:

$$\Rightarrow \log \boldsymbol{I}_{\text{out}} = \boldsymbol{N}\boldsymbol{v}$$

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$$\log \boldsymbol{I}_{\text{out}} = \boldsymbol{N}\ \boldsymbol{v}$$

output images — matrix of voxel-pixel contributions — voxel densities

$\Rightarrow$ Linear in v, simply solve it

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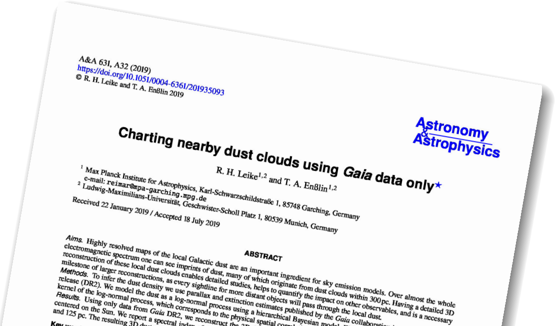
A&A 631, A32 (2019)
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Astronomy & Astrophysics

Charting nearby dust clouds using *Gaia* data only★

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ABSTRACT

Aims. Highly resolved maps of the local Galactic dust are an important ingredient for sky emission models. Over almost the whole electromagnetic spectrum one can see imprints of dust, many of which originate from dust clouds within 300 pc. Having a detailed 3D reconstruction of these local dust clouds enables detailed studies, helps to quantify the impact on other observables, and is a necessary milestone of larger reconstructions, as every sightline for more distant objects will pass through the local dust.
Methods. To infer the dust density we use parallax and extinction estimates published by the *Gaia* collaboration in the second data release (DR2). We model the dust as a log-normal process using a hierarchical Bayesian model. ... kernel of the log-normal process, which corresponds to the physical spatial correlation ...
Results. Using only data from *Gaia* DR2, we reconstruct the 3D ... centered on the Sun. We report a spectral index ... and 125 pc. The resulting 3D dust ...

1.2 With Emission

Now every point in space also emits light $g(d)$
Usually dependent on $\tau(d)$: $g(d) := c(d)\tau(d)$ for some color $c(d)$.

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$$I_{\text{out}} = T(t_{\text{max}})I_{\text{in}} + \int_0^{t_{\text{max}}} g(t)T(t)dt$$

$$\text{with } T(s) = \exp\left(-\int_0^s \tau(v(r(t)))dt\right)$$

This is a nested integral and cannot be solved analytically.

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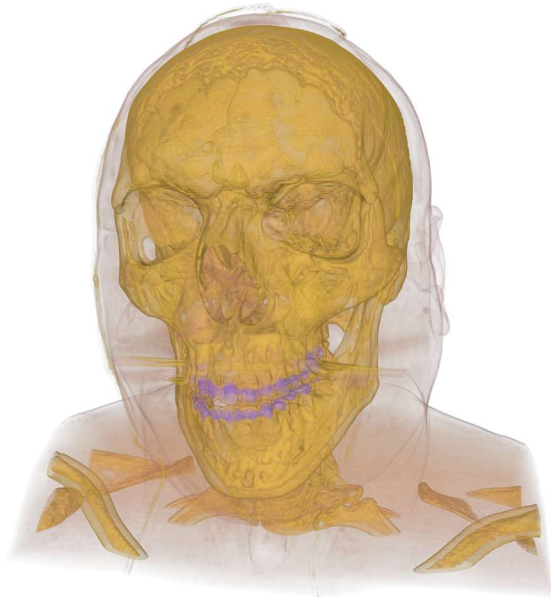

$\tau(d), c(d)$ arbitrary

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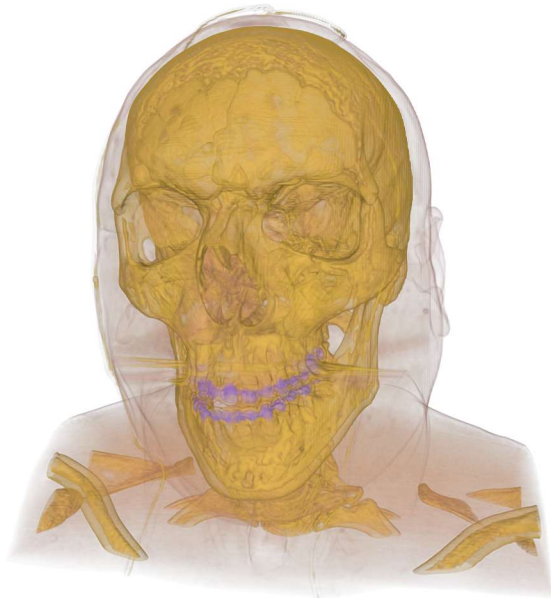

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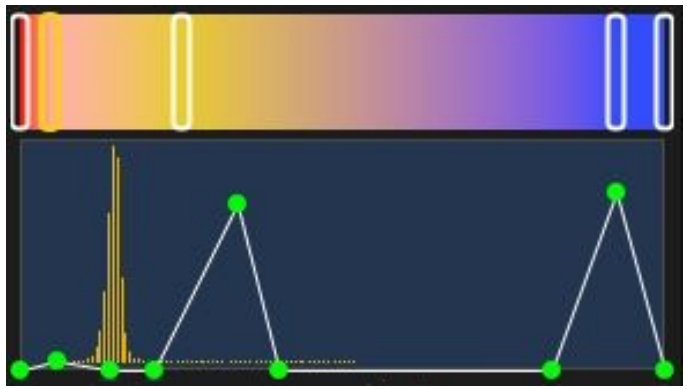

2. Automatic Differentiation

$$\boldsymbol{x}_0 = \text{const}$$
$$\boldsymbol{x}_1 = f_1(\boldsymbol{x}_0, \boldsymbol{w}_1)$$
$$\boldsymbol{x}_2 = f_2(\boldsymbol{x}_1, \boldsymbol{w}_2)$$
$$\dots$$
$$\boldsymbol{x}_k = f_k(\boldsymbol{x}_{k-1}, \boldsymbol{w}_k)$$

with

$k \in \mathbb{N}$	number of operations
$\boldsymbol{x}_i \in \mathbb{R}^n$	state variables
$\boldsymbol{x}_k \in \mathbb{R}^m$	size of the output / last state
$\boldsymbol{w}_i \in \mathbb{R}^p$	parameters

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\end{aligned}
\qquad \Rightarrow \frac{\mathrm{d}\boldsymbol{x}_k}{\mathrm{d}\boldsymbol{w}_1}
$$

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Replace each $\boldsymbol{x}_i, \boldsymbol{w}_i$ by

$$\tilde{\boldsymbol{x}}_i = \left\langle \boldsymbol{x}_i, \frac{\mathrm{d}\boldsymbol{x}_i}{\mathrm{d}\boldsymbol{w}_1} \right\rangle, \quad \tilde{\boldsymbol{w}}_i = \left\langle \boldsymbol{w}_i, \frac{\mathrm{d}\boldsymbol{w}_i}{\mathrm{d}\boldsymbol{w}_1} \right\rangle$$

Note that $\tilde{\boldsymbol{x}}_0 = \langle \boldsymbol{x}_0, 0 \rangle, \tilde{\boldsymbol{w}}_1 = \langle \boldsymbol{w}_1, 1 \rangle$

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Replace a function $\boldsymbol{c} = f(\boldsymbol{a}, \boldsymbol{b})$ by $\tilde{\boldsymbol{c}} = \tilde{f}(\tilde{\boldsymbol{a}}, \tilde{\boldsymbol{b}})$ with

$$\left\langle \boldsymbol{c}, \frac{\mathrm{d}\boldsymbol{c}}{\mathrm{d}\boldsymbol{c}_1} \right\rangle = \left\langle f(a, b), \right.$$
$$\left. J_{f|a}(a, b) \frac{\mathrm{d}\boldsymbol{a}_i}{\mathrm{d}\boldsymbol{w}_1} + J_{f|b}(a, b) \frac{\mathrm{d}\boldsymbol{b}_i}{\mathrm{d}\boldsymbol{w}_1} \right\rangle$$

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$\Rightarrow O(kn^2 + knp)$ computations

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Assume $x_k \in \mathbb{R}$ is a scalar ($m = 1$), define

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\hat{\boldsymbol{x}}_i = \frac{\mathrm{d}x_k}{\mathrm{d}\boldsymbol{x}_i} \in \mathbb{R}^n, \quad \hat{\boldsymbol{w}}_i = \frac{\mathrm{d}x_k}{\mathrm{d}\boldsymbol{w}_i} \in \mathbb{R}^p,
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For DVR we need some hybrid method...