Funding Public Projects: A Case for the Nash Product Rule

Florian Brandl [1] Felix Brandt [2] **Matthias Greger** [2]
Dominik Peters [3] Christian Stricker [2] Warut Suksompong [4]

[1]University of Bonn

[2]Technical University of Munich

[3]University of Toronto

[4]National University of Singapore

The 17th Conference on Web and Internet Economics
December 15, 2021

- Annual Charity Matching Programs of companies
  - In 2021, Microsoft employees raised $208 million for 27,000 nonprofits and schools.[1]
  - Since 2011, Apple's Employee Giving Program has donated nearly $725 million to 39,000 organizations.[2]

- Employees donate independently of mutual interests.
  - Employee 1 would like to donate to Greenpeace ( ) or WWF ( ).
    Employee 2 prefers to donate to or Unicef ( ).
    An efficient distribution rule would allocate both contributions to .
  - Employees can benefit from coordinating the donations.

Which distribution rule should we use?

[1] https://www.microsoft.com/en-us/corporate-responsibility/philanthropies/employee-engagement

[2] https://www.apple.com/newsroom/2021/12/apple-marks-a-year-of-giving-in-the-communities-it-calls-home/

Main Results

Which distribution rule should be chosen?

- Goal:
  - Guarantee (Pareto-)efficiency of the distribution.
  - Incentivize agents to donate to maximize the gains from coordination.
    $\rightarrow$ requires a strong participation axiom as contributions are initially *owned* by the agents.
- The **Nash product rule** is the only distribution rule we are aware of that simultaneously satisfies efficiency and such a strong participation axiom.

Model

- Set N of agents with contributions $C = \{C_i\}_{i \in N}$ not exceeding the individual budgets $\{B_i\}_{i \in N}$.

- Set A of projects the agents can contribute to:

Model

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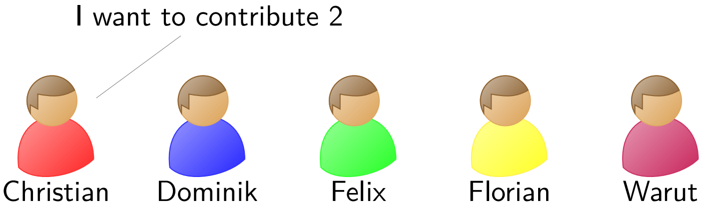

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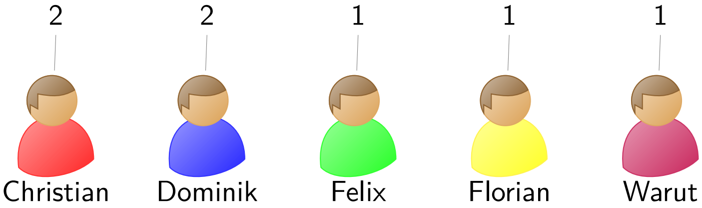

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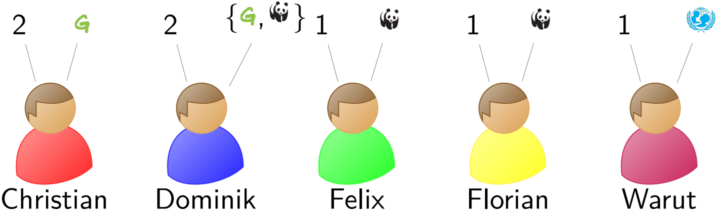

- Set A of projects the agents can contribute to: .
- Individual utility functions $u_i : A \to \mathbb{R}_{\geq 0}$ (here: $\to \{0, 1\}$).
  $\to$ value for one unit that is allocated to project x.
- Distribution $\delta : A \to \mathbb{R}_{\geq 0}$ with $\sum_{x \in A} \delta(x) = \sum_{i \in N} C_i = |C|$.
  $\to u_i(\delta) = \sum_{x \in A} \delta(x) u_i(x)$.
- Distribution rule f determining the returned δ.

The Nash Product Rule

Agents	C_i				$u_i(\delta)$
Christian	2	·	·	·	·
Dominik	2	·	·	·	·
Felix	1	·	·	·	·
Florian	1	·	·	·	·
Warut	1	·	·	·	·

Which distribution should be chosen?

The Nash Product Rule

Definition: Nash Product Rule

For an arbitrary profile C,

$$NASH(C) = \underset{\delta \in \Delta(|C|)}{\arg\max} \prod_{i \in N} u_i(\delta)^{C_i}.$$

Agents	C_i				$u_i(\delta)$
Christian	2	2	·	·	3
Dominik	2	1	1	·	6
Felix	1	·	1	·	3
Florian	1	·	1	·	3
Warut	1	·	·	1	1
δ_{NASH}	7	3	3	1	16

✓ efficiency

$\rightarrow$ no $\delta' \in \Delta(|C|)$ s.t.
$u_i(\delta') \geq u_i(\delta)$ for all $i \in N$ and
$u_i(\delta') > u_i(\delta)$ for some $i \in N$.

The Nash Product Rule

Agents	C_i				$u_i(\delta)$
Christian	2	2	·	·	3
Dominik	2	1	1	·	6
Felix	1	·	1	·	3
Florian	1	·	1	·	3
Warut	1	·	·	1	1
δ_{NASH}	7	3	3	1	16

- Observation: This distribution can be decomposed into individual distributions such that each agent only contributes to his approved projects.
- We call such distributions **decomposable**.
- Decomposability becomes very important when the distribution rule only gives recommendations to the agents.
- A NASH distribution δ can always be decomposed via $\delta_i(x) = C_i \frac{u_i(x)}{u_i(\delta)} \delta(x)$.

The Nash Product Rule

Definition: Nash Product Rule

For an arbitrary profile C,

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Agents	C_i				$u_i(\delta)$
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✓ efficiency

✓ decomposability

→ Already sufficient to ensure participation?

The Nash Product Rule

Agents	C_i				$u_i(\delta)$
Christian	2	2	·	·	3
Dominik	2	1	1	·	6
Felix	1	·	1	·	3
Florian	1	·	1	·	3
Warut	1	·	·	1	1
δ	7	3	3	1	16

Agents	C_i				$u_i(\delta')$
Christian	2	2	·	·	2
Dominik	2	·	2	·	5
Felix	0	·	·	·	3 +1
Florian	1	·	1	·	3
Warut	1	·	·	1	1
δ'	6	2	3	1	14

- By not participating, i.e., saving his contribution, Felix can increase his utility gains $(u_i(\delta) - C_i)$ from coordination.
- Goal: Contributing the entire budget should be a dominant strategy for each agent.
  $\rightarrow$ captured by the axiom of **contribution incentive-compatibility**.

The Nash Product Rule

A mechanism f is **contribution incentive-compatible** if for each $i \in N$ and all profiles C, $u_i(f(C_{-i}, C_i)) - C_i$ is weakly increasing in C_i.

Agents	C_i				$u_i(\delta)$
Christian	2	2	·	·	3
Dominik	2	1	1	·	6
Felix	1	·	1	·	3
Florian	1	·	1	·	3
Warut	1	·	·	1	1
δ_{NASH}	7	3	3	1	16

Agents	C_i				$u_i(\delta)$
Christian	2	2	·	·	$10/3$
Dominik	2	$4/3$	$2/3$	·	5
Felix	0	·	·	·	$5/3$ $+1$
Florian	1	·	1	·	$5/3$
Warut	1	·	·	1	1
δ_{NASH}	6	$10/3$	$5/3$	1	$38/3$

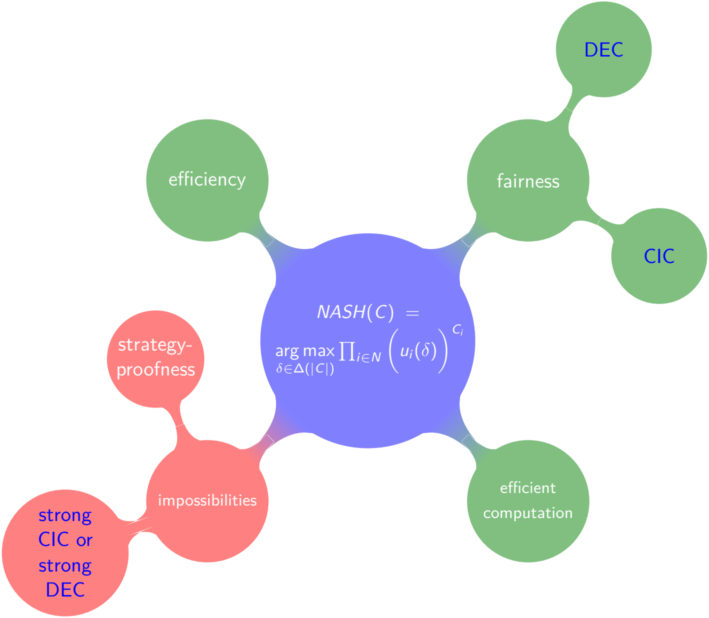
Nash Product Rule

Theorem

The Nash Product Rule satisfies efficiency, decomposability and contribution incentive-compatibility.

We are not aware of any other distribution rule that satisfies efficiency AND contribution incentive-compatibility!

Summary and Further Remarks



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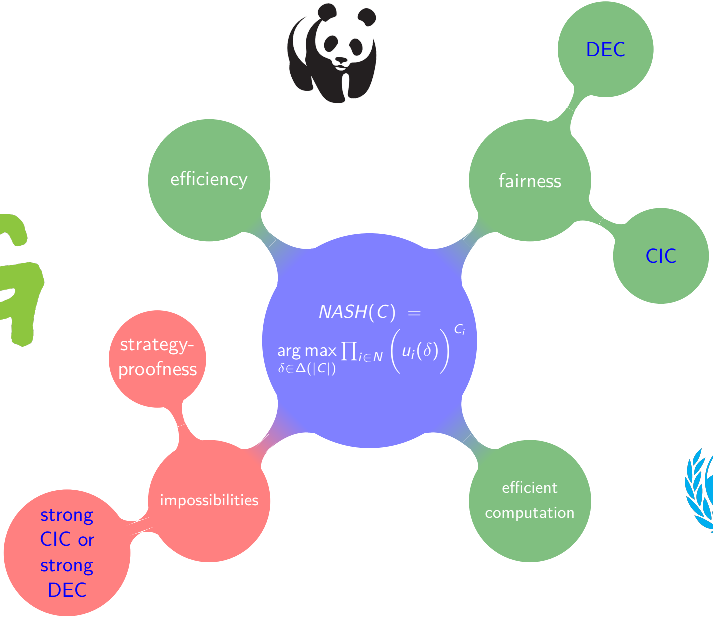

$NASH(C) = \arg\max_{\delta \in \Delta(|C|)} \prod_{i \in N} \left(u_i(\delta) \right)^{C_i}$
efficiency
fairness
DEC
CIC
efficient computation
impossibilities
strategy-proofness
strong CIC or strong DEC

References

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